THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XLVI May Number 5 1953

Fewer Teachers to Meet Greater Demand

By RAY C. MAUL

Research Division, National Education Association, Washington, D.C.

ARE WE on the eve of a shortage of qualified mathematics teachers to staff the classrooms of the high schools of the nation? Is the annual supply of new candidates coming from the college graduating classes no longer equal to the demand? Have the rapidly changing conditions brought us face to face with a shortage of adequately prepared teachers?

The findings of the 1953 national teacher supply-and-demand investigation indicate affirmative anwsers to these questions. No longer can we be complacent about the problem which is already critical at the elementary-school level. Superintendents now searching for mathematics teachers to enter service next September are discovering a change from a reasonably good supply of interested candidates to a shortage of persons qualified for this important assignment. The trend in the foreseeable future is unmistakably toward an even more unfavorable ratio of supply to demand.

What has brought about this change? Only a few years ago the number of available qualified candidates seemed equal to the needs in all high-school fields, except possibly the vocational subjects. Increasing attention has been focused upon the quality of the high-school curriculum in the confidence that fully prepared staff members would be available to go forward with experimentation and to carry out changes. If a shortage of personnel appeared, it was assumed to be no more than local.

CHANGES HAVE ESCAPED NOTICE

Changed conditions have come upon us so rapidly that they are not commonly understood. First, since the bulk of new teaching candidates comes from the annual groups of college graduating classes, these facts stand out:

1. The total number of college graduates has decreased each year since 1950.
2. The total number of college graduates prepared to enter high-school teaching has decreased each year since 1950.
3. The total number of college graduates prepared to teach mathematics has decreased each year since 1950.

Second, the fields of employment open to college graduates have been steadily increasing in number in recent years. The college degree is becoming a requirement for more and more jobs. Personnel directors of many types of corporations are now familiar figures on college campuses at this time of the year. The continuance of this trend may be expected.

Third, increased emphasis on national defense since 1950 has placed a special premium on the college graduate with major training in mathematics. The demand for persons trained in the various scientific fields parallels the search for such personnel during the war. There is

nothing in the current scene to indicate a diminution of this emphasis. On the contrary, the place of America among the nations points up the growth of the demand for scientifically trained leadership. Since sound instruction in mathematics undergirds the work of every kind of scientific practitioner, the role of the mathematics teacher takes on added significance.

Fourth, and perhaps most critical just now, is the growing probability that every able bodied young man must anticipate at least two years of military service. The prospects for the indefinite continuance of both a hot and a cold war give this factor new meaning. Heretofore we Americans have met a threat to our national safety with an all-out effort, but always in the confidence that we were meeting a temporary emergency, and that we would, of course, demobilize when it had been met. Now we face the prospect of maintaining a large standing force for an indefinite period.

A New Era Impends

The fifth change in the current scene—perhaps the greatest factor in the supply-and-demand problem—is the impending change in the size of the high-school student body. We have come along for so many years with a constant number of students in attendance that it is hard to realize the high school is about to move into a new era. September 1953 brought the first slight upturn in total high-school enrollment in many years. By 1955 this increase will be widespread; by 1958–1960 it will be in full swing. This vast expansion in numbers to attend high school creates a need of new dimensions for qualified teachers. Where two high-school students were present in 1950, there will be three in 1960.

The change from a high-school student body of 6 million to one of 9 million is neither mere speculation nor a temporary affair. More children were born in 1952 than in any previous year. They will not attain normal ninth-grade age until 1966; they must not be expected to leave the high-school student body until 1970. At this moment there is no indication that the number in attendance will decrease even after that date. Thus the staff needs for the next two decades must be anticipated in these new terms.

The Immediate Outlook

A quick look at Table 1 shows that the supply of qualified candidates coming from the colleges is in a downtrend. As World War II closed, the total college enrollment, while absorbing the accumulated backlog and under the stimulus of the G. I. Bill of Rights, grew at an unprecedented rate. In 1946 the total number of bachelor's-degree candidates was 136,000. Four years later, in 1950, this figure had zoomed to 434,000. In 1950, the peak year, no less than 4,618 college graduates with full majors in mathematics were eligible to standard high-school teaching certificates. Graduates of that year will remember that there was no oversupply of candidates for the teaching positions then open to them.

Table 1.—Total Number of College and University Graduates Prepared to Teach in High School, with the Number and Per Cent of Total Prepared to Teach Mathematics

Year	Number of graduates prepared to teach in high school	Number of graduates prepared to teach mathematics	Per cent of total
1	2	3	4
1949.....	66,890	3,684	5.51%
1950.....	86,890	4,618	5.31
1951.....	73,015	4,118	5.64
1952.....	61,510	3,142	5.11
1953.....	55,468	2,710	4.89

Since 1950 the number of college graduates prepared to teach mathematics has steadily decreased, along with the general decrease in the total number of college

(*Continued on page 310*)

Insurance Looks Ahead*

By E. J. Faulkner
Woodmen Accident Company, Lincoln, Nebraska

Scarcely any adult American can be oblivious of insurance. In one way or another it touches us in every facet of our daily living. By the very fact of our being we are subject to many hazards. In order to appreciate the place that insurance occupies in our economic order, it may be well to note the personal risks and the business risks which each of us runs. Personal risks are of two kinds—the risk of loss of income and of loss of property. Loss of income to an individual or family may result from death, from permanent or temporary disability, from unemployment or superannuation. With the exception of unemployment, this category of risks is the field of life, health and accident insurance. It protects against the financial consequences of dying too soon, of living too long, or of loss of earning capacity through disability. In addition to these loss-of-income risks, to greater or lesser degree each of us runs the risk of property loss through destruction or damage by fire or lightning, water leakage, earthquake, explosion, riot and civil commotion, burglary, theft or robbery and forgery. We run the risk of property loss due to liability for injury caused to the person or property of another by our own negligence or that of our agent where the injury is to the person of an employee or of a stranger or where the injury is to the property of another. In addition to these loss-of-income and property risks which we run as individuals, we may be subject to the business risk of loss of capital or income from several other causes such as the peril of transportation of goods on land and sea or in the air; hail, frost or other cause of crop failure; defalcation or other misconduct of employees; failure of contractors to fulfill contracts; failure of depositories or unusual credit losses on sales. All of these property risks are the field of the fire, causalty, marine and fidelity insurance companies.

From tiny beginnings, often in ancient times, insurance in its various manifestations has grown to be a mighty safeguard protecting the members of our society from the multitude of hazards which threaten them and their property. To give you some notion of the extent of the insurance operations in the United States, it may be noted that in the year 1951, fire and casualty insurance companies collected $5,720,000,000 as the premiums for the protection that they issued. Accident and health insurance companies wrote premiums in excess of $1,750,000,000 as the consideration for the protection that they provided. Under their contracts, accident and health companies and other voluntary insurance associations protected more than 86,000,000 Americans against hospital costs and more than 40,000,000 Americans against loss of income due to disability. Life insurance in force in the United States last year exceeded in face amount $253,000,000,000, providing a large measure of protection to more than three out of four of all American families. This is an average of $5,500 of life insurance protection per family. The services of the institution of insurance in providing protection against loss, in assisting individuals and families to accumulate savings, in providing loss prevention service and in supplying capital funds with which to energize and expand our capitalistic economy are so wide-spread and diversified as to defy adequate description in the few moments available to us today. I have cited these figures simply to give you some notion of the all-pervasive nature of

* Presented at the luncheon meeting of the 13th Christmas meeting of the National Council of Teachers of Mathematics, Lincoln, Nebraska, December 31, 1952.

the institution of insurance and of its continuing and intimate importance to each one of us.

Knowing this, it must be particularly gratifying to you as teachers of mathematics to recognize the complete reliance of this great industry on mathematical principles and techniques. The insurance function is a risk bearing function. The insurance company or insurer is the economic mechanism that has been devised for spreading the risk. Risk has been defined as the chance of loss and insurance as the substitution of a small certain loss, the premium, for the risk of a large, uncertain loss, the contingency insured against. Through insurance a large number of persons, all subject to the same risk, band together to share the losses incurred by the unfortunate members of the group. This sharing or pooling is illustrated in a crude way by one of the earliest examples of marine insurance. The olden Chinese merchants, bringing their goods from the interior down the broad Yangtze River faced with trepidation the possible overturning of their boats in the rapids of that river. It became their custom to unload their boats at the head of the rapids and then reload with each boat bearing part of the cargo of every other merchant. In this way if a single boat was lost, each shared in the loss and no one suffered from the destruction of an entire cargo.

In all of the centuries' long development of insurance, the underwriter consciously or otherwise has applied the Law of Large Numbers. The ancient Phoenicians in their first attempts at marine insurance; the early Egyptians, Chinese and Hindus with their benevolent societies, and the members of the Roman Collegia, forerunners of life insurance companies, did not have the benefit of precise dissertations on the Theory of Probabilities, but they knew enough of the stability of mass phenomena from trial and error to recognize the importance of spread of risk to any successful insurance undertaking. It is the operation of the Law of Large Numbers that takes the insurance transaction out of the sphere of gambling or speculation. The tendency of mass phenomena toward regularity produces the relative certainty that is essential to insurance as distinct from the element of chance which gives gambling its appeal. That this distinction was not generally understood resulted in the prohibition of certain types of insurance at various periods in history. For instance, life insurance was forbidden in many European countries during the Middle Ages. Some remnants of this lack of understanding of the insurance function carried over into the recent past of our own country among religious sects that held it ungodly to insure the human life value.

As we have learned more about mathematics and more about the many aspects of economic risk, it has been possible for enterprising underwriters to develop new kinds of insurance in response to the developing needs of the public. For instance, one of the most spectacular developments of the past twenty years has been the growth of hospital insurance. With the depression, when hospital endowments began to disappear and the hospitals were for the most part in severe financial straits, insurance underwriters and hospital administrators devised the plan of pre-paying hospital costs as a means of providing the insuring public with a needed protection and at the same time alleviating the financial problems of the hospitals. It will be of particular interest to you that one of the very first voluntary hospital insurance groups was organized among the school teachers of Dallas, Texas. From a very humble beginning just about twenty years ago, hospitalization insurance has expanded in this country until today well more than half of our entire population owns this kind of protection.

Underwriters and mathematicians generally consider that a risk is insurable if it has the following characteristics: (1)

the hazard must be accidental, that is the loss must be neither necessary nor impossible; (2) the hazard must be sporadic; (3) it must be expected neither too frequently nor too rarely; and (4) it must be statistically measurable and computable or at least estimable. These certainly are characteristics that must evoke definite and familiar impressions in the mind of the student of mathematics. They confirm the statement heretofore made that the entire insurance operation has its bedrock foundation in the science that you teach.

In the routine operation of an insurance company, mathematics plays its most significant role in two ways: first, as a tool for forecasting insurance cost and expense, in other words, in the calculation of premium rates; and second, in recording the result of the insurance operation through the maintenance of necessary records and reserves and experience data. These two roles are obviously closely intertwined. The pioneers of each type of insurance, lacking a background of experience on which to base their premium estimates, had to resort to pure judgment in establishing the first rates. Today we have the benefit of a vast array of mechanical equipment and well-tried methods for recording, reproducing, and arranging our experience data. It is exceedingly significant, however, that no inventor has yet been wise enough to produce a machine that will think. True, we are beginning to see some of the wonders of electronics when applied to office methods. I am sure that you will all recall the amazing demonstration of the Univac Electronic Computer which did such an amazing job of pointing to election trends the evening of November 4, 1952. But even with the unfolding marvels of electronics we should recognize that no matter how perfect the method, how complicated the machine, nor how well-proved the theory, the end result will be no better than the judgment of the person who is making the calculation or reasoning through the problem. Quite recently I was much impressed with the comment of Professor Ralph Blanchard, Chairman of the Department of Insurance, School of Business, Columbia University, who said:

> The insurance business has done itself a great disservice by creating the impression that it is highly scientific, that from its accumulated statistics it can produce an accurate prophecy of its future losses by expert application of mathematical formulae. The public has heard much of mortality tables, statistics of experience, the Law of Large Numbers, the Theory of Probability, and actuarial mathematics, mostly in praise of the wonder-working attributes of these tools in the hands of insurance technicians. Too little has been heard of the qualifying words and phrases that are applicable to the use of the tools; of credibility, "conditions being the same," trends, emergencies, extraneous influences.

We have but to look at the history of insurance to know the importance of qualifying our forecasts. The life insurance industry has been the beneficiary of an increasingly favorable trend in mortality for more than two hundred years. Yet even the life insurers have had their troubles, as witness the unfavorable effect of improvement of mortality on annuity experience or the inability of the life insurance actuary to foresee and prophesy the marked decline of return on investment capital during the last twenty years or the serious death toll due to the influenza epidemic of 1918 and 1919. These examples are cited not because they threatened the stability and soundness of the institution of life insurance, but simply to emphasize that even in the most highly developed and scientific area of all insurance operation, reliance cannot be placed solely on mathematical calculation without adding a large corrective factor for underwriting judgment. Professor Blanchard goes on to say that

> If this situation is true of life insurance, where the questions of amount of loss, and whether and to whom it is payable are usually easily and definitely answered, what of lines of insurance that are subject to a multitude of causes affecting both basic perils and resultant losses? I need not, nor would I have time to, detail for this group the factors that influence the

occurrence and amounts of losses. But let me remind you that among them are business conditions, legislative enactments, availability of materials, weather, engineering, organized pressures, political expediency, even the procreative tendencies of workers and the remarriage proclivities of widows.

Again may I say that what I am attempting to suggest is that insurance operations today depend as they always have on a high degree of keen underwriting judgment and business acumen even when buttressed by a vast fund of loss experience. This, of course, is so because the great difficulty in using the experience of the past is to know its reliability as a basis for predicting the future. Judgment must be applied to the past if we attempt to penetrate the fog of the future and judgment of itself has a way of resting on the past and of being warped by bias.

Insurance looks ahead to expanding spheres of greater usefulness. As our society becomes more complex with increasingly urban type of living, we become more and more interdependent, less able to assure security for ourselves or to bear alone the risks to our person and our property. We become more and more reliant on the insurance mechanisms of society to assure us the security that we need and to provide a stimulus to our capitalistic economy from which flows our high standard of living. To a future that promises to be increasingly complex, you as teachers of mathematics have the rare opportunity and privilege of making one of the greatest of all possible contributions. Yours is the opportunity of developing an orderly type of mind in the young people who will be our business and political leaders of tomorrow. We need the expert technicians, but even more as a people we need the ability to think straight. We need more and more of a logical approach to the problems of life and living. We need the inquiring type of mind that does not fall prey either to tricks of logic or to the blandishments of the demagogue. Specifically, in the business of insurance we need more and more men and women who can look at a balance sheet or a page of statistics without fear or confusion. The crying need is not for the Ph.D. who can navigate unerringly through the mathematical stratosphere, but rather for the man or woman who is well grounded in arithmetic, algebra and business statistics. I do not mean in any way to cast reflection upon the genius of mathematics, the man or woman who has the rare ability to open up whole new horizons in the research laboratory or at the engineer's table. I speak simply of the mass need for men and women in business and every-day life who have developed orderly thinking processes and who have a good basic understanding of mathematical fundamentals. In directing the minds of young Americans along these lines, you teachers of mathematics will be insuring a prepetuation of the type of free enterprise economy that has raised America to a standard of living heretofore unknown in the history of mankind.

Fewer Teachers to Meet Greater Demand

(*Continued from page 306*)

graduates. The 1953 group promises to include only 2,710. This is a drop of 43.7 per cent from the high of 4,618 in 1950. It must be noted, also, that 1,790 men are in this group of 2,710 anticipated mathematics majors graduating this spring with preparation to teach in high school. It cannot be determined at this time how many of these men will enter military service.

In the fall of 1952 the general college enrollment gave the first indication of reversing the downtrend of the past four years. The modest increase (1.5 per cent) was entirely in the freshman class. Juniors and sophomores—the classes of 1954 and 1955—are fewer than a year ago. This means that there is little prospect of increasing the number of available qualified candidates in the near future.

The Case for the Syllogism in Plane Geometry

By James F. Ulrich
Arthur Hill High School, Saginaw, Michigan

It is generally agreed that one of the chief objectives of plane geometry is to foster reflective thought in the mind of the pupil. At various times and in various places it has been clearly stated that the achievement of this objective demands a clear and explicit treatment of the components of reflective thought itself, if the desired benefits from transfer of training are to accrue. The writer will make no attempt to reconcile the many forms of the term "reflective thought"; this has been done elsewhere.[1] Whenever the elements of reflective thought are listed in a discussion of the objectives of plane geometry, one of those elements will be similar to this: *Plane geometry should help the student to understand the nature of and develop an appreciation for deductive proof.* Few would deny that this is an important mission of geometry. It is the approach made by the teacher in presenting the deductive proof to which the remainder of this article is devoted.

Since the syllogism constitutes the heart of deduction, how can its omission be justified? If an understanding of deduction is our aim, how does the average tenth grader see much connection betweeen deduction and a direct geometric proof when deduction *itself* is not explicitly treated? It is believed by the writer that the syllogism[2] should be used as an introduction, first to deduction, and then to the deductive geometric proof.

Over ten years ago Rolland R. Smith thoroughly explained how students under his direction showed improvement in the understanding of a geometric proof when careful attention was given to the syllogism.[3] The conclusions reached by Smith as a result of his study are well worth reviewing by every geometry teacher.

The remainder of this article contains an outline of the presentation the writer has used in this phase of the course. The reader may feel that the approach to be described is too formal to be worthwhile if used in the early weeks of the course. If that is the case, then some of the material could be presented informally near the beginning of the course and the rest later. Much of the following material will seem exceedingly elementary, but it is this very simplicity which helps to make the over-all picture meaningful to the student.

Introducing the Syllogism

Any discussion of syllogisms necessarily involves logic because it is the field in which the syllogism plays a very important role. Formal logic is an exciting yet a rigorous study. Tenth grade pupils should be allowed a share of the excitement while they are spared the rigors. The geometry teacher who intends to have sufficient background for conducting a clear approach to the deductive proof through the syllogism should be aware of the fundamentals of logic—especially those which pertain to the various types of syllogisms.

The point is that the teacher should know more about the syllogism than he attempts to teach. He must select from the

[1] R. E. Pingry, "Critical Thinking—What Is It?" The Mathematics Teacher, XLIV (November 1951), 466–470.

[2] Formal logic usually treats the following five types of syllogisms: (1) categorical, (2) hypothetical, (3) conjunctive, (4) alternative, (5) disjunctive. When the term "syllogism" is used from this point on, unless specifically stated as otherwise, it will refer to the first two types because they are more closely related to the present topic.

[3] Rolland R. Smith, "Three Major Difficulties in the Learning of Demonstrative Geometry." The Mathematics Teacher, XXXIII (March 1940), 99–134 and XXXIII (April 1940), 150–178.

vocabulary of the logician those terms he intends to have his pupils use. Through the use of many of the terms and processes of formal logic the effect of the whole presentation most assuredly would be aborted.

For this reason, in the following paragraphs, many of the terms of formal logic are employed for the sake of brevity and clarity to the reader, but those which are intended for the ears of the pupil are printed in **bold type.**

Simple, valid, categorical syllogisms can be presented first which will demonstrate the form of the **syllogism** to the pupil. The pupils will readily contribute examples for discussion and thus the teacher will be able to explain gradually the properties of the syllogism. The syllogism may be defined broadly at first as "a series of three **statements,** the first two of which are called **premises** and the third, the **conclusion.** The premises must lead to the conclusion." The parts are then given more careful attention and are introduced as:

The **General Statement** (abbreviated **G**)
The **Specific Statement** (abbreviated **S**)
The **Conclusion** (abbreviated **C**)

The nature of the premises is considered next. The "statements" referred to a moment ago are now treated as **propositions.**[4] The four types of categorical propositions and the traditional forms in which they are found are shown below. (Where "*X*" is an individual person, thing, or relationship.)[5]

TYPE	FORM
1. Universal-affirmative	All _____ are _____ .
(a) General	
(b) Singular	*X* is a _____ .
2. Universal-negative	No _____ are _____ .
(a) General	
(b) Singular	*X* is not a _____ .
3. Particular-affirmative	Some _____ are _____ .
4. Particular-negative	Some _____ are not _____ .

From this chart it is seen readily that the most frequent application for the geometric syllogism uses premises of the type 1(a) and 1(b).[6] For instance:

Example #1

G. All triangles with two equal sides are isosceles. 1(a)
S. *RST* is a triangle with two equal sides. 1(b)
C. Triangle *RST* is isosceles.

A second illustration would be:

Example #2

G. All vertical angles are equal. 1(a)
S. Angle 1 and angle 2 are vertical angles. 1(b)
C. Angle 1 equals angle 2.

The terms "general statement" and "specific statement" used by the pupils for 1(a) and 1(b) respectively serve the purpose quite well for all syllogisms discussed, even though they may have inaccurate implications when propositions of the type 2, 3, or 4 are brought up.

Up to this point **valid** syllogisms have been used, but now the **invalid** syllogism may be introduced and the syllogisms

[4] A proposition is defined as a statement that is either true or false.

[5] This chart is adapted from: Lionel Ruby, *Logic, An Introduction.* New York: J. B. Lippincott Company, 1950, p. 200.

[6] General statements in the "if-then" form are preferred for use in geometry. They will be explained subsequently, but provide no serious objection to the above remark.

must be judged valid or invalid in accord with the laws prescribed for validity. Euler's circles, which are **circular diagrams,** can be used to test for validity. The approach can be broken down into that of (1) diagramming individual propositions, and then (2) diagramming the two propositions in a syllogism which constitute the premises. The logician tests for validity with his rules concerning the distribution of terms, but the geometry student can use these circular diagrams effectively for the situations with which he will be confronted.

The hypothetical proposition may be treated merely as a proposition in the **if-then form;** that is, a statement containing an "if" or conditional clause and a "then" clause or conclusion. The propositions of geometry used as reasons in a direct proof are more desirable when in the form of the hypothetical proposition.[7] The hypothetical proposition does not complicate the picture thus far depicted for the simple reason that categorical propositions can be translated into hypothetical propositions which have the **same meaning,** and conversely. The circular diagrams may still be used for if-then propositions after they have been changed to equivalent categorical statements. Practice in translating hypothetical to categorical propositions is given, followed by practice in diagramming hypothetical propositions.

The "mixed" hypothetical syllogism is the one met in plane geometry. It is composed of a **major premise** in the if-then form, a **minor premise** that is a categorical proposition, usually of type 1(b), and a categorical conclusion. (All conclusions in this discussion are categorical.)

Since the if-then major premise may be translated into an equivalent general statement, this second type of syllogism offers no new obstacles whatsoever to the student, in the light of what he has learned about the categorical syllogism. However, it is now possible to test for the validity of the hypothetical syllogism without resorting to the categorical proposition and the circular diagrams. The following rules provide this test: (1) the specific statement must fulfill *all* of the conditions in the if-clause of the general statement, and (2) the conclusion must repeat the conclusion of the general statement with respect to the particular person or thing named in the specific statement.[8]

As various syllogisms are presented to the pupils it is vitally important to point out the difference between valid conclusions and **true** conclusions. Examples can easily be given in which one or both of the premises are false and yet the conclusions reached are valid. Such syllogisms aptly demonstrate that conclusions are true only within the limits of the premises from which they are derived.

The term **deduction** itself has not yet been defined. If defined now it should have greater meaning to the student. The definition itself is quite simple: deductive reasoning is the process of arguing from given premises to their necessary conclustions.[9]

The comments made thus far are deemed by the writer to be what could be called "minimum" requirements for a sound understanding by the pupil of the nature of the syllogism and the properties of deduction. Before the transition is made to the direct proof in geometry, it is highly recommended that additional exercises involving syllogisms be discussed in the classroom. Such work with syllogisms may consist of (1) a discussion of categorical types other than the universal-affirmative, and (2) practice with the enthymeme.

The universal-negative categorical prop-

[7] This view has been expressed many times by geometricians. See: H. Sitomer, "If-Then in Plane Geometry." THE MATHEMATICS TEACHER, XXXI (November 1938), 326–329.

[8] Smith, *op. cit.*, April, 1940, p. 177.

[9] Deductive conclusions may be "mediate" or "immediate." Conclusions of syllogisms are the result of mediate inference. In geometry, examples of immediate inferences are the converses of definitions and the contrapositives of theorems. In such cases, no intervening premise is present from which to derive the conclusion.

osition is especially important because it involves the principle of **"class exclusion."** This principle can be easily illustrated with Euler's circles, and is a valuable concept in the realm of reflective thought.

An enthymeme is merely a syllogism with one or more of the three parts (major premise, minor premise, conclusion) omitted. There are four kinds: (1) major premise omitted, (2) minor premise omitted, (3) conclusion omitted, and (4) minor premise and conclusion omitted. The fourth kind is not very common. Time devoted to a discussion of the enthymeme is well rewarded indeed, for the enthymeme provides the basis for making the shift to geometric applications.

Great strides can be made in understanding the syllogism when the various forms of the enthymeme are discussed. Examples can and should be drawn from non-geometrical sources, especially the field of advertising.[10] Here the detection of implied assumptions, an important concept of critical thought, can be practiced by the pupils. Actually the enthymeme need not be treated as a separate phase of this unit for at almost any stage beyond the introduction of the syllogism, the student should be required to supply missing premises or conclusions.

The Syllogism in the Geometric Proof

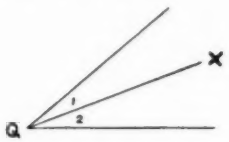
The stage has now been set for the entrance of the formal geometric proof. Comprehension of the *meaning* of a geometric proof can be achieved when it is written in two ways in this introductory period. First, the proof is written in syllogistic form; then it is put in the customary geometric form. When the student sees both ways of writing the proof, he can note the similarities and the differences. The example which follows will illustrate:

[10] For an excellent source of material of this type, see: Gilbert Ulmer, *Some Suggestions for Teaching Geometry to Develop Clear Thinking*, Monograph, University of Kansas Publications, Kansas Studies in Education, Vol. II, No. 7, 1949, pp. 2–20.

Example #3

Given: Angle Q, and QX the bisector of angle Q.

Prove: Angle 1 = angle 2



Syllogistic Proof:

G. If a line is the bisector of an angle, then it divides the angle into two equal parts.

S. QX (a line) is the bisector of angle Q.

C. Angle 1 equals angle 2.

Geometric Proof:

S. 1. QX is the bisector of angle Q. (Given.)

C. 2. Angle 1 = angle 2. (G. If a line is the bisector of an angle, then it divides the angle into two equal parts.)

The similarity between the two is that every part of the syllogistic proof is present in the geometric proof. The differences are: (1) the geometric proof requires a reason for each step. (2) the general statement of the syllogism is merely located in a different place, i.e. opposite its conclusion, and (3) each step and reason is given a number. The abbreviations G, S, and C are used as indicated for they are a constant reminder of the presence of the syllogism.

Example #3 brings out two additional points worthy of mention. The student must supply the general statement for this proof, but note that in doing so he is merely providing the missing major premise of a syllogism. He already has practiced doing the *same* thing in his work with the enthymeme! Secondly, once he realizes what the general statement should be, he has his choice of the two forms of that general statement; either the one shown above or its converse. Since he

knows that his selection must render a valid syllogism, he is able to check his choice to make sure he has the correct form of the definition. Naturally the reasons he seeks for the right column in his geometric proofs will not always be definitions, but when they are definitions or general statements (the converses of which have been accepted or proved) he has a definite tool for dealing with them. He has established that tool through his study of the syllogism.

After a few two-step geometric proofs have been discussed and the student has practiced writing them in both syllogistic and geometric forms, he then can be introduced to the geometric proof with more than two steps. Such proofs of course involve chains of syllogisms, and the process of writing the proof in both syllogistic and geometric form is continued.

Example #4

Given: $XY \perp YB$
$AB \perp YB$
Prove: Angle Y = angle B

Syllogistic Proof:

G_1. If two lines are perpendicular, then they form right angles.
S_1. $XY \perp YB$, $AB \perp YB$.
C_1. Angle Y is a right angle. Angle B is a right angle.
G_2. If angles are right angles, then they are equal.
S_2. Angle Y and angle B are right angles.
C_2. Angle Y equals angle B.

Geometric Proof:

S_1. 1. $XY \perp YB$, $AB \perp YB$. (Given.)
C_1S_2. 2. Angle Y is a right angle. Angle B is a right angle. (G_1. If two lines are perpendicular, then they form right angles.)
C_2 3. Angle Y = angle B. (G_2. If angles are right angles, then they are equal.)

The salient feature here is that the conclusion of the first syllogism (C_1), having been proved, now serves as the specific statement (S_2) for the second syllogism.[11] When the proof involves two or more syllogisms, similar transformations take place regularly.

Occasionally, the specific statement for a syllogism must be provided by the figure under consideration. For example, if the definition of adjacent angles is involved, in order to establish the fact that "two angles with a common vertex and a common side between them" actually have that relationship to each other, the figure itself must be referred to.

When an approach such as this is made to the geometric proof, the student sees from his syllogisms what the order of steps should be, why certain steps and reasons must be included, why others are unnecessary; in short, he is able to check the proof for accuracy and validity himself. After sufficient practice has been given in this "double" form of proof, the geometric form alone may be required.

Conclusion

It is the contention of the writer that the teacher should frequently point out just where and how the syllogism occurs in the direct proof. To do this, the syllogism itself and its fundamental properties can be introduced and explained; then the all-important transition from the syllogism to the geometric proof should be *explicitly* demonstrated. The difference between a geometric proof and a syllogistic proof is merely one of form, but that difference should be thoroughly discussed.

(*Continued on page 325*)

[11] The use of a system of abbreviations such as these, with subscripts, is recommended by H. Fawcett and M. L. Hartung in: *The Forty-fifth Yearbook of the National Society for the Study of Education*, Part I, "The Measurement of Understanding," Chicago: The University of Chicago Press, 1946, p. 169.

High School Algebra for Bright Students*

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IT HAS been assumed too frequently that the modernized college preparatory or "academic" curriculum in mathematics is suitable for talented students. Relatively little thought and investigation have been devoted to their needs. As a result of this situation, some educational leaders believe that this group is being inadequately served. The group of pupils includes most of the potential leaders of the next generation. It is not undemocratic to single them out for special attention. On the contrary, the preservation and perfection of the democratic way of life require the full utilization of all the human resources of the nation. This means that the brilliant as well as the dull student should have an opportunity to work and develop to his full capacity.

THE OBJECTIVES

Assume that a group of ninth grade students of well above average learning ability is available. Every student in the group expects to enter college. Their parents and the colleges expect them to know some algebra, and in the school they attend the major portion of this learning must take place in the ninth grade. What sort of an algebra course should they be offered? A partial answer to this question may be found in the *Second Report of the Commission on Post-War Plans* of the National Council of Teachers of Mathematics. Thesis 13 of the Commission reads as follows: "In most schools first-year algebra should be evaluated in terms of good practice."[1] In the discussion of this thesis it is pointed out that first-year algebra has been greatly improved during the last quarter of a century. Seventeen desirable trends are listed, including among others the following implied recommendations:

(1) Reduce the manipulation of symbolism (nests of parentheses, four-story fractions, involved cases of factoring, difficult cases of simultaneous equations, etc.);
(3) Emphasize the notion of dependence or functions;
(4) Teach with great care the meaning of a formula;
(5) Apply graphic techniques widely;
(10) Use a more sensible program of "drill" based on the fact that a pupil learns more quickly and remembers longer the things that he understands fully;
(11) Use a few simple interesting, and practical applications to motivate each new principle and topic;
(12) Strive to improve the problem material by selecting functional applications (aviation, the school shop, general science, etc.).

Certainly a course for superior students should be expected to bear some resemblance to these suggestions. It is to be noted that they do not call for more than is now recognized as "good practice," and thus make no pretense of pioneering. Other answers to the question raised above, but with much the same import, may be found elsewhere. Representative of the best of these suggestions are those of W. D. Reeve in an article published in 1943.[2]

It has long seemed to the writer that it is possible and desirable to include most recommendations of this type under one

* This paper, under a different title, was read at a mathematics conference at Teachers College, Columbia University, in July, 1947.

[1] The Second Report of the Commission on Post-War Plans, THE MATHEMATICS TEACHER, XXXVIII (May, 1945), 207–8.

[2] William David Reeve, "A Proposal for Mathematics Education in the Secondary Schools of the United States." THE MATHEMATICS TEACHER, XXXVI (January, 1943), 11–20.

central idea or objective; namely, the student should begin to learn some of the mathematical methods of treating quantitative data as they are used by mature investigators. This means that he should understand how data are acquired by measurement, experiment, census, and other means. He should understand some of the ways of analyzing, interpreting, and summarizing the data and the relationships implicit in them. This includes the analyses of tabular representations, graphical interpretations, symbolic representation of relationships by formulas, and finally, the utilization of formulas and algebraic technique to discover and analyze other relationships implicit in the data and which would otherwise remain hidden or obscure. These objectives emphasize the aspects of algebra which are dominant in the applications. Elementary instances of those aspects which appeal most to the student of modern pure algebra are not to be ignored, but it is much harder to defend them in the secondary school curriculum, even when only the brighter students are being considered. The remander of this discussion will be devoted to a description of a course which put more emphasis than is common upon the objectives just outlined. It will be helpful, however, to provide first some background data about the situation in which the course was given.

The Students

The course to be described was given to twenty-five students in Grade IX of the Laboratory School of the University of Chicago. The class met for forty-five minutes Monday, and for fifty minutes on Tuesday, Wednesday, and Thursday. The class did not meet on Friday, and a number of days were given to taking tests in connection with other research going on in the Laboratory School. Approximately 125 class meetings were devoted to mathematics. During the Spring Quarter, when much of the work to be described was being done, the class was taught by a practice teacher without prior experience, and with only occasional comments or reemphasis by the writer.

The mean of the Binet intelligence quotients of this group was 141.4. The lowest was 120, the highest 184, a range of 64. The standard deviation was about 15, and two-thirds of the intelligence quotients fell within 15 points of the mean. Although this was a select group, the range of learning ability, as measured by the Binet test, was about as great as that found in many "normal" groups. Scores on arithmetical achievement, reading rate and comprehension were correspondingly high and wide in range. There were 16 boys and 9 girls in the group. Except for their high intelligence-test scores, they were a normal, well-adjusted, but unusually lively bunch of boys and girls. There were many other boys and girls in their ninth grade class who could have qualified for the group, but who were not included. The students were not selected on the basis of unusual ability in *mathematics*. This class-group had been formed at the seventh grade level in connection with a study by the department of English, and except for a few pupils who had been transferred or added, they were held together as a group for three years in mathematics, but not in other subjects.

In the seventh grade the work of these pupils emphasized informal geometry. Apart from this emphasis and the methods of treatment used, the program was normal except for one additional feature. For several years the mathematics department in the Laboratory School had been experimenting with the grade placement of some of the work in statistics. As seventh graders most of these pupils were exposed to the unit on *Statistics* by Schorling-Clark-Lankford, and they did commendable work on it.

In the eighth grade this class was allowed to move as fast as they could, but were not forced. During the first semester they were able to cover as much of the usual eighth-grade program as was con-

sidered valuable at that level. During the the second semester, they began the study of so-called "First Year Algebra," and easily succeeded in doing a normal half-year of work. This practice of beginning algebra in Grade VIII is not uncommon in private secondary schools.

The Course

When school opened in the autumn of 1946, the first unit of work studied by the group in Grade IX was on simultaneous linear equations. After three weeks they were given a test on first year algebra and the decision was made to continue the study of algebra through the remaining important topics of the modern standard course. Between October 15 and the Christmas holidays the group studied in the main the following topics: special products and factoring, square root and radicals, and quadratic equations (with emphasis on completing the square). These topics were all treated thoroughly from the point-of-view of understanding and meaning, but the complexity of the forms used was restricted to types which commonly arise in the solution of problems. Algebraic fractions were treated very superficially and the same criterion was used; namely, the examples were no more complex that those needed in the solution of realistic verbal problems. From time to time other materials, chiefly applications, were introduced for the sake of enriching the conceptual framework, and not for the purpose of increasing technical skill.

At the conclusion of this work the students were eager to undertake the study of plane geometry. This was considered unwise for several reasons. First, it would throw them too far out of step with other ninth graders. There was no guarantee that they would be held together as a group another year, and if in the autumn of 1947 they were to be distributed among classes starting demonstrative geometry, either the task of the teachers would be made unduly difficult in order to care for them properly, or they would be allowed to mark time and become bored while their classmates caught up with them. Second, the position was taken that acceleration was not to be sought for its own sake, but rather that these pupils were to be kept learning at reasonably full capacity. Since there was much still to be learned through a reorganized approach to algebra, they were persuaded to undertake it. Immediately following the Christmas holidays the class began the study of progressions. This was followed by a thorough treatment of exponents and the binomial expansion for any exponent, logarithms and a brief introduction to the slide rule. Throughout all of this work certain aspects of the function concept were emphasized, and graphical representation was used extensively. Practical applications were kept in the limelight, particularly those drawn from the mathematics of investment which employ the progressions and binomial expressions in their treatment; e.g., compound interest and annuities. Approximately two months were used for this portion of the program, and about three months remained to round out the treatment of the function concept and some of its applications in scientific work.

The systematic treatment of the function concept which closed this course followed an outline which is essentially the same as that which has been frequently discussed in the literature of mathematical education.[3] It was organized in terms of functional types of increasing complexity ranging from the linear from $y=ax$ to the exponential $y=ab^x$. The chief unusual feature was the emphasis which was put upon curve-fitting, including determining the formula of a given type which would give the "best fit" in a certain restricted sense which will be explained later.

Since the function concept had not been neglected in earlier portions of the course,

[3] *Mathematics in General Education*, Report of the Committee on the Function of Mathematics in General Education of the Commission on Secondary School Curriculum (New York: D. Appleton-Century Co., 1940), 148–59, 396–97.

part of this work constituted an informal review of earlier learning. Linear functions of the type $y=ax$ and $y=ax+b$ were studied from the point of view of the table, graph, formula, and verbal statement, and were so treated that generalizations of the following sort emerged:

1. Constant differences in the independent variable are accompanied by constant differences in the dependent variable.
2. If the scales are uniform and the same unit is used on both axes, the graph is a straight line through the origin, or through $(0, b)$; the slope is determined by the parameter a, and the inclination of the line to the X-axis may be judged in terms of the value of a.

Next, tables of experimental data involving observational errors were introduced. It was clear from the graph and from the approximately constant first differences in the table that a formula of the type $y=ax+b$ might give a satisfactory representation of the data, and the problem of determining the parameters a and b to secure the "best-fitting" formula of this type was undertaken. The practical source of these data was the time-worn example of the extension of a spring under various weights. Seven pairs of observations of the length l as a function of the weight w were available. The substitution of these pairs in the form $l_0=aw+b$ yielded seven linear equations in a and b. These provided 21 *combinations* of two linear equations which when solved produced 21 pairs of values of a and b, or 21 possible formulas. It was obvious to the students that some of these did not make a very good fit. Each formula was used to secure a table of *computed* values of l_c as a function of w, and the differences l_0-l_c were also computed. The graphical interpretation of these deviations was stressed, and it was then decided that the formula which yielded the minimum sum of the absolute values of the deviations would be considered the best among those found. It happened that several of the formulas yielded the same sum. This led to the squaring of the deviations and the use of the "minimum sum of squares" as the criterion.

Finally, the 21 equations for a and b were separated into two groups which were summed as polynomials. The resulting two equations were solved for a and b and the testing for "best fit" repeated. The students were thus given an introduction to the "method of averages." The analogy between what they were doing and the procedures of mature scientific investigators was continuously stressed. It was necessary to study several examples in this way before the ideas began to be fully understood.

The remainder of this discussion can be somewhat briefer, since essentially the same methods were used for other types of functions. The quadratics $y=ax^2$, $y=ax^2+b$, and finally $y=ax^2+bx+c$ (which was converted by completing the square to the form $y=a(x-h)^2+k$) were studied until generalizations of the following sort emerged:

1. If the scales are uniform, the graph is a curve (called the parabola) with a line of symmetry parallel to the Y-axis and through $x=h$. The vertex is at (h, k).
2. The parameter a determines the apparent "shape" of the curve, acting as a scale factor. All such parabolas may, however, be drawn with a single pattern (made from the curve of $y=ax^2$) by properly changing the scales on the axes, and properly locating the vertex.
3. If the scales are not uniform, the graph will not appear to be a parabola. In particular, if a non-uniform scale of squares is used on the y-axis, the graph of $y=x^2$ will appear to be a straight line.[4]

[4] Cf. Maurice L. Hartung and Robert L. Erickson, "Graphical Methods in Science and Mathematics Teaching," *School Science and Mathematics*, L (March, 1950), 200–8.

4. If the differences in the independent variable are constant, corresponding first differences of the dependent variable are in arithmetic progression, and *second* differences are constant.

After studying and interpreting a number of practical examples of this form, data on the stopping distance of a car as a function of speed were given, and it was decided to fit them with a formula of the type $D=aV^2+bV$. The procedure outlined above in connection with the linear function was repeated to so determine a and b as to obtain a "best-fitting" curve and formula in the senses explained above. The value of having such a formula was discussed, including such notions as its possible uses for interpolation, extrapolation, and for determining V as a function of D by "changing the subject" of the formula.

In much the same way the circle, ellipse, and hyperbola were studied. For example, a circle was fitted to data on the "Distribution of Income According to Population—1929," as suggested by Rosander.[5] In studying the ellipse, an equation like $4x^2+9y^2=36$ was put in the form $y=\pm(2/3)\sqrt{9-x^2}$, and the graph sketched by drawing the circle $y=\pm\sqrt{9-x^2}$ and then locating points whose ordinates were each 2/3 of the corresponding ordinates of the circle. This illustrates a commonly neglected type of functional thinking.

The exponential function was introduced by making a table for the number of ancestors in each of several generations back. In this case the formula $A=2^x$ may be fitted by inspection. The graphs of several such functions of the form $y=ab^x$ were drawn, using first uniform scales, then plotting log y against x on uniform scales, and finally plotting y against x on semi-log graph paper. To provide data for curve-fitting, Newton's Law of Cooling, which is of the exponential type, was used, and the "best-fitting" curve and formula for a set of data were obtained.[6]

The Evaluation

To close this paper without some comments on the evaluation of achievement would leave certain questions unanswered. First, the students were for the most part highly interested and enthusiastic about their work. Discussion was lively and usually to the point.

Second, the students were tested on their understanding of empirical equations by the use of questions of the following type:

1. State the exact linear relation between L and w from the following experimental data:

w	2	4
L	14	21

2. The graph of the exponential equation $y=10^{ax}$ is a straight line if y is plotted on a _____ scale and x is plotted on a _____ scale.
3. Name at least two methods for finding the best empirical equation between two variables to fit observed data. Which of your methods do you consider more accurate?
4. If you have a set of observed data for two related variables, how do you determine the *general type* of the equation that best fits the data?
5. What is the purpose of finding empirical equations? What can they be used for after they are determined?

Scores on the test for "Ideas About Empirical Equations" were expressed in per cent correct, and the median for the group was 85. The students unquestion-

[5] A. C. Rosander, "Mathematical Analysis in the Social Sciences," The Mathematics Teacher." XXIX (October, 1936), 289–94.

[6] Cf. L. Hogben, "Mathematics for the Million" (New York: W. W. Norton and Company, 1937), 419–20.

ably made considerable growth, and probably have a better understanding of the way mathematics is often used in scientific investigations than many teachers acquire under existing teacher-training programs. There were, however, no norms or other bases of comparison available for this test, and no further statistical study of these results was made.

The evaluation data which are most dependable were obtained by administering standard algebra tests. It should be noted that these students had not taken the sort of course for which the tests were originally validated. Many topics treated in the usual first year and intermediate algebra courses had been omitted or slighted, and others not covered by the test were introduced and emphasized. There were no systematic reviews. Concepts and skills which were not automatically maintained through *use* in the course were allowed to take care of themselves. Thus at best these test data merely indicate the extent to which these students were able under these circumstances to acquire the concepts and skills normally acquired in first year and intermediate algebra.

After three weeks in Grade IX the pupils were given the Cooperative Elementary Algebra Test. The mean of scaled scores was 64.2, which is at the 88th percentile of norms based on scores from 130 public schools. The interpretation of this is that the average score of the group exceeded the scores of 88 per cent of the 15,000 pupils who had studied algebra one year and whose scores were used to prepare norms. The lowest scaled score of 50 was at the 38th percentile; the highest scaled score of 76 was at the 99th percentile. During the last week of the course this test was administered again to determine if there had been any loss or gain from the instruction. One of the best students was absent, but the mean scaled score of the rest was 70, which is at the 96th percentile of the norms. The difference between the means of the scaled scores is significant at less than the 1 per cent level. Thus it appears that the students, in spite of high initial achievement, made a definite gain in the achievement of those objectives of first year algebra measured by the Cooperative Elementary Algebra Test.

During the last week of the course the Cooperative Intermediate Algebra Test was also administered. One of the above-average students had already left school and was not tested. The median of the scaled scores of the remainder of the group was at the 62nd percentile. The lowest scaled score of 45 was at about the 8th percentile, the highest scaled score of 76 was at the 98th percentile. These percentile norms are based on scores of 4000 students in Grade XI in 60 schools. It is to be noted again that much of the usual content of intermediate algebra had been omitted. There was no work on factoring, algebraic fractions, and similar topics which occupy the early weeks of the traditional course in intermediate algebra. Linear systems in more than two unknowns were not treated formally. The theory of quadratic equations and equations of higher degree was omitted. Only the barest essentials of radicals were treated. Yet these bright ninth grade pupils got enough out of what was treated to do commendable work on a reputable standard test of intermediate algebra.

The conclusion that seems warranted on the basis of this experience is that bright students can, at an early age, learn much more mathematics than they are usually given an opportunity to study. The hypothesis that as a rule they are not being fully challenged has received some additional experimental verification. There is no evidence that the material these students were taught was the best for them at their level of advancement, nor that other students of lower ability could succeed with the same sort of work. As a matter of opinion only, it appears to the

(*Continued on page 325*)

Let's Do Something for the Gifted in Mathematics

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FOR DECADES educators have insisted on the importance of providing an adequate educational program for the gifted. However, the application of this program in the mathematics classroom has usually been neglected or sporadic. The increasingly critical need for trained scientists no longer permits us to be lackadaisical in using a most precious human resource, the mathematically gifted student.

Descriptions of the gifted always emphasize their possession of characteristics such as intellectual curiosity, ingenuity, independence, high reading interest, imagination, creative talent, leadership and ability to assimilate and generalize. In order to capitalize on these unique traits, the program for the gifted should provide a variety of enrichment activities. A supply of mathematics books and periodicals should be available to provide literature to satisfy intellectual curiosity and reading interests. Creative talent and independent work habits may be developed through research projects, written reports, or the construction of mathematical devices. The possibilities of using imagination in dealing with infinity, the fourth dimension, non-Euclidean geometry, a new number base, or Boolean algebra are tremendous. The analysis of original data collected at school or in the community can be a vehicle by which social responsibility and leadership may be awakened as well as a means of applying methods of analysis. To capitalize on the ability of the gifted to assimilate and generalize difficult material, it is possible for the teacher to direct the student to a much more intensive coverage of a given course. The study of graphing can be extended to topics in analytical geometry, the logic of geometry to statistical inference, the study of variation to elements of calculus. This differentiation of assignment is probably the simplest and often the most practical method of providing for the superior student.

Another method of offering an enrichment program is to provide an enriched curriculum. This affords opportunities for the gifted to carry extra subjects during the school day, during night school, or through correspondence courses. Individualized instruction or group instruction within the class are methods advocated in the literature but seldom practiced by the over-burdened teacher.

One of the most common proposals has been to provide special classes for the superior student. These classes permit the extensive study of mathematical topics adapted to the ability of the student and presented at a challenging rate. It provides an organization in which acceleration, enrichment and independent activities can be coordinated into a balanced program. In the small school it has been adapted in the form of clubs or seminars, often meeting after school. We use a tremendous amount of out-of-school time, provide special coaches, and buy all kinds of equipment for the athletically gifted without too much concern for conceit on the part of stars or frustration on the part of the third stringer. Let's begin working for equivalent provision for the mentally gifted!

Another frequent proposal is to accelerate the superior student. This method has the advantage of providing challenging material at a pace that does not permit poor work habits and avoids the frustration of lock-step class work. It involves certain risks of social and emotional maladjustment. However, several studies have shown that moderate acceleration is desirable when the gifted individual is socially and emotionally mature.

The desire for social leadership has often

resulted in the gifted becoming so involved in extra-curricular activities that he lacks time to build his foundation in science and mathematics. Social responsibility and community leadership must be assured the gifted. But it should be part of a balanced program planned with the help of a well informed counselor or teacher. This planning may involve the selection of goals as well as the building of interests. It should also explore means of financial support if it is needed to attain the goal selected. Counselors can also be helpful in establishing a testing program that will identify the gifted, locate their strengths and weaknesses, and measure the progress they are making.

However, all of these methods will fail unless the gifted has been motivated to extend himself beyond the ordinary requirements of the classroom. The stimulation of an enthusiastic, competent teacher, the recognition of high accomplishment, and the influence of a satisfying school environment are indispensable to maximum achievement.

An ancient tale relates how a scholarly deliberation over the number of teeth in a horse's mouth was disrupted by a young friar. He proposed to solve the problem by finding a horse and looking in its mouth. In our present dilemma we may do a similar thing by interviewing gifted individuals of varied ages. Here are some of their responses to the questions as to what methods, materials or factors were most influential in their high school mathematics classes:

"It wasn't the frosting i.e. puzzles, tricks, or unusual problems that stimulated me, it was the inherent meaning of the process or rule."

"The dullness of my mathematics instruction made me resolve to become a teacher of mathematics so that others wouldn't have to live through the same experiences."

"I worked for good grades but the ease of getting an A gave me a false measure of success."

"My teacher was the key to my interest and attainment in mathematics. She stimulated me and then gave me the opportunity to pursue my interests independently."

Here is the report of an individual who captured all the awards given by his high school and entered college at 15 years of age.

"I was dropped from algebra at the end of six weeks because I persisted in wanting a definition of a minus number. To my lasting regret this has closed the doors to science and mathematics and logic. I have maintained my status through verbalism. I want the mathematics teachers of my children to provide a stimulating emotional environment, to introduce the tools of analysis, to build an appreciation of the aesthetic; factual knowledge will take care of itself."

"Grades were not a stimulus for me because the grade I would get was always obvious, but receiving a letter for continuous appearance on the honor roll was a tremendous incentive. It compensated for my inability to get an athletic letter."

"I enjoy mathematics because it is easy for me and gives me a feeling of accomplishment. I don't want to accelerate for two reasons: (1) it won't stick with me if I do, (2) I like to have time to help others in the class."

"I liked the competition with other class members especially when we worked independently with a chart of the progress of each individual. I would like a book with practical applications like *One, Two, Three, . . . , Infinity* by George Gamow (Viking Press) as a second text book."

"The bright spot in my high school mathematics was the teacher who fostered independence and self-study."

"My achievement was largely due to the stimulation of my parents and my brothers. My parents insisted that I had to do as well as my brother. Actually, I got a lot of satisfaction out of beating another student whom I didn't like and appearing on the honor roll."

It is noteworthy that these persons place great emphasis on motivation, independence, and search for meaning.

To get a further look into the horse's mouth, a questionnaire was sent to leaders in mathematics education. Here are the replies from 10 persons in 8 states:

1. What do you consider the most effective means of providing for the gifted?

Mathematics clubs
Correspondence courses
Special projects and problems
Some work in helping other students
Time for individual conferences
Individualizing instruction
Special training for teachers who are to teach such students
Different and more effective instructional materials
A testing program that measures growth
Special courses
Instructors who have a broad background of mathematics and related fields and an understanding of the gifted so as to be able to select proper materials

(Note that no one mentioned acceleration or motivation.)

2. What materials of instruction do you consider most effective in providing for the gifted?

The same materials available for good instruction but use them a much shorter time.
Stimulating subject matter which challenges the gifted
Good textbooks
Supplementary booklets
Readings and problems from books such as *Mathematics and the Imagination*
Library materials for special reports
Articles and sections of THE MATHEMATICS TEACHER

(No one mentioned models, excursions, community surveys, laboratory projects, recreational materials.)

3. What activities, projects, etc. do you consider most effective in providing for the gifted?

Mathematics contests and contest teams
Assembly programs
Projects dealing with applications and vocational use
Supplementary topics such as the duodecimal system
Activities which demand much productive thinking i.e. planning, executing, evaluating
Readings in mathematics literature
Challenging mathematics problems
Projects involving abstract thinking
Formal research through reading and mental analysis
Making models to demonstrate a mathematical concept
Preparation of a topic for the Science Talent Search, Pepsi-Cola contest

4. What research is being done in your state on the needs of the gifted or on ways of providing for these needs?

No research was reported to be in progress. However, Kenneth E. Henderson of the University of Illinois is working on a project which is attempting to find out how students' ability to think critically and logically may be improved through science, mathematics, social studies and English. The project is just beginning to get under way by working with high ability classes.

In the light of these comments it would seem that it is essential that we take steps such as the following:

1. *Arouse a new sense of responsibility for providing for the needs of the gifted.* Parents, students, teachers, newsmen, government officials, businessmen, scientists, and ordinary citizens should be informed of the need for conserving this indispensable human resource. One of the best places to begin the building of an attitude of respect for excellence of performance is in our classrooms which contain the future parents, voters, and leaders of

our society. If states can afford multi-million dollar projects for the mentally deficient, we can also afford special educational opportunities for the gifted.

2. *Provide teachers with the materials, facilities, and time needed for an effective enrichment program.* This involves not only reading material, but also space for group work, facilities for creative activities, and time for individual guidance. Most mathematics classrooms are too small, too barren, too crowded for a stimulating program, a condition which is being perpetuated in new buildings.

3. *Provide teachers who have the requisite ability, education, and interest to meet the challenge of the gifted student.* This means an educational background that includes intensive study of mathematics and science, extensive work in other fields and special training in teaching the superior child. It may mean that the teacher of the gifted should also be a gifted individual if he is to understand their needs. It is not a task to be assigned to a teacher as a reward for many years of service in the school but rather to the person with the enthusiasm, ingenuity, and interest needed to build an effective program.

4. *Provide the gifted student the necessary opportunities and financial support needed to complete his educational background.* This may involve scholarships or jobs for financial support, community activities which develop a sense of responsibility and offer leadership activities, facilities such as libraries, museums, summer programs which provide opportunities for creative activities and social adjustment.

5. *Establish a school organization, curriculum, evaluation program, and counseling services that will provide the facilities for carrying out an effective program.*

6. *Finally, organize a research program that will discover our needs and evaluate the effectiveness of a variety of methods of meeting this problem.* This research program should be a broad coordinated project sponsored by professional organizations and financed by government, foundation, or private funds.

This is indeed a major project for our nation. However, it is an urgent project that needs to be started immediately if we are to retain our world leadership. It must have support from every segment of our society and that includes you!

Syllogism in Plane Geometry

(*Continued from page 315*)

A rather detailed discussion has been given here for the geometry teacher interested in using such a presentation. The writer believes material of this type can be beneficial in giving the student a better appreciation for deduction in general, and in particular, for the deductive geometric proof. The logical structure of geometry has been praised for centuries. Perhaps an approach of this nature can aid in making this logical structure more apparent to the average tenth grade student.

High School Algebra

(*Continued from page 321*)

writer that students of sufficient ability to warrant their taking algebra at all can do most of the work described above if suitable materials and methods are used and a much slower pace is set. In closing, perhaps this experiment affords some evidence for the validity of the following sentence from a paper by Harold Hotelling:

> The pettifogging insistence on small things, on drill, chewing away on elementary practice in place of rapid progress into higher mathematics is really the great obstacle to development in a great many people of a higher mathematical culture.[7]

[7] Harold Hotelling, "Some Little Known Applications of Mathematics," THE MATHEMATICS TEACHER, XXIX (April, 1936), 157–69.

Using Recreational Mathematics Materials in the Classroom

By Louis Grant Brandes
Encinal High School, Alameda, California

The value of recreational mathematics has long been recognized by the mathematics teachers. The variety and inherent interest in recreational mathematics items can be utilized to improve pupil attitudes and to make learning more effective. Some teachers have found such items a useful device in overcoming emotional blocks to learning. The pupil who finds an interest in the recreational side of mathematics acquires a permanent source of enjoyment. The teacher who has materials available in this area has resources for making classwork more effective.

One limiting factor in the classroom use of mathematical recreations in the secondary schools has been the difficulty in locating suitable materials. Some teachers have been able to collect a usable supply over a long period, but the prospect is discouraging to a beginning teacher.

In an effort to gain more information concerning the use of recreational mathematics materials by teachers, this writer conducted a questionnaire study. A questionnaire was mailed to fifty persons chosen at random from a list of a hundred persons who had requested and received copies of selected recreational mathematics materials. The materials consisted of a collection of short objective tests, number oddities, tricks, optical illusions, mathematical facts, and stories of interest to pupils in grades seven through twelve.

The questionnaire was directed toward determining how recreational mathematics materials were used by selected persons in teaching secondary school mathematics subjects. It sought to ascertain the effect of using the materials upon pupil interest and classwork, the ways in which the materials were used in teaching pupils, the grade level and type of classes in which the materials were used, and teacher reactions and opinions toward the use of such materials.

The study was limited in that it consisted largely of the reactions and opinions of persons with sufficient interest in the use of recreational materials to request copies of such materials after reading reports in educational periodicals.[1] These reactions and opinions, however, are believed significant in that they indicate how some mathematics teachers feel they have been able to improve pupil attitudes and make learning more effective in their classes.

Of the fifty copies of the questionnaire forwarded, forty-two were returned; thirty-two were from within the state of California, while one each was received from ten different states. Thirty-one of the returns were from teachers of secondary-school grades; eleven returns were from persons other than teachers in the secondary schools. Of the thirty-one returns from secondary-school teachers, twenty-eight were from teachers in the public schools, while three were from teachers in private and parochial schools. Persons other than public secondary-school teachers returning copies of the questionnaire included two principals, one assistant principal, four school librarians, two college professors, and two college students.

Twenty-nine of the thirty-one responding teachers considered that the use of recreational mathematics materials in the teaching of their mathematics classes had improved their classes. Most of the re-

[1] L. G. Brandes, "Math Can Be Fun; Tricks, Puzzles, Wrinkles," *The Clearing House*, XXV October, 1950, pp. 75–79.

L. B. Kinney, "Book Reviews," *California Journal of Secondary Education*, December, 1951, pp. 502–3.

sponding teachers felt that the recreational mathematics materials stimulated the interest of most of the pupils in their classes; some of the teachers expressed the opinion that the materials stimulated, either directly or indirectly, the interest of the entire class.

Attempt was made through questionnaire items to determine in which grades and subjects (of the grades and subjects taught by responding teachers) recreational mathematics materials were most frequently presented. According to the responses to these items by the thirty-one responding teachers, recreational mathematics materials were offered in the mathematics subjects for all secondary grades, seven through fourteen, but were offered by the highest proportion of teachers in the ninth and tenth grade mathematics subjects. These subjects included, in order of frequency of response, general mathematics, elementary algebra, and plane geometry. The responses indicated that a smaller proportion of the teachers presented recreational mathematics materials in the subjects of advanced algebra, solid geometry, and trigonometry; these subjects appeared to receive the more formal teaching. Responses to further inquiry indicated that recreational mathematics materials were frequently used in mathematics classes for remedial, mentally handicapped, and gifted children as well as in mathematics classes for regular pupils.

Asked, "How do you use recreational mathematics materials in teaching your classes?" twenty-nine of the thirty-one responding teachers gave replies. In order of the frequency of mention, the following ways of using recreational mathematics materials were offered:

1. Materials presented to the entire class as a part of a daily lesson plan.
2. Materials presented in class to individuals and small groups to stimulate interest in mathematics.
3. Materials presented to individual pupils in class who have completed their assignments.
4. Materials presented to an entire class to work on outside of regular class time, such as in study hall or at home.
5. Materials placed on chalk board before classes assemble, to gain the attention of pupils and "settle" the classes.
6. Materials presented to individual pupils, who have expressed an interest in recreational mathematics, to work on outside of regular class time.
7. Materials placed on a bulletin board in the classroom and changed at regular intervals.
8. Materials presented during "fill in" time, usually at the end of a period.
9. Materials presented to classes on special days, such as the days before school holidays, visiting days, etc.

Other ways in which teachers reported using recreational mathematics materials include presentations for recreational and group meetings, items for school paper, items for school bulletin boards to stimulate interest of pupils not enrolled in mathematics classes, materials for mathematics club activities, and materials for rainy day sessions.

Another of the questions asked the teachers was, "Do you have any evidence to support the opinion that classwork was or was not improved through the use of recreational mathematics in your classes?" Ten of the responding teachers replied YES, twenty-two replied NO, while three did not reply to this item. Representative of the evidence presented by the ten teachers replying YES are the following excerpts:

> "The pupils began to organize the fundamentals of mathematics and to show that simple concepts are transferable to the field as a whole. It seemed an excellent link between numbers and literal problems."
>
> "Stimulation of interest resulted in

pupil improvement in handing in their assignments on time."

"Pupil interest stimulated through recreational mathematics led to the development of projects that involved comprehensive study."

"Among other things, recreational mathematics tends to stimulate com petition among the better pupils. They will work hard to outdo each other to get the solution first. Also, there is a noticeable effort on the part of the pupils to find such material in outside sources to bring to class to 'stump' others."

"This type of mathematics has been a stimulation to many pupils. When they saw how some of the 'trick' problems could be solved by algebra, for instance, they applied such methods to other problems."

"Pupils began to see in their own way a unifying thread running through the subjects; they became conscious of the many uses of mathematics not otherwise noticed by many of them."

"Through presenting recreational problems in connection with specific skill to be developed, even the poorest pupils tried all the homework problems and often brought new problems to class."

"There was an obvious change in attitude of the pupils toward mathematics; from work to fun."

The teachers were asked to give their reactions and to make comments concerning their experiences with recreational mathematics materials in the teaching of mathematics subjects in the secondary school grades. Twenty-six of the thirty-one teachers returning the questionnaire responded to this item. The following excerpts from their remarks express the "feelings" indicated by the responding teachers:

"It is a great aid in exciting and maintaining interest in courses of mathematics."

"Recreational mathematics items tend to throw more life and informality into what might otherwise become a formal and drab subject. All of my pupils 'go for it.' "

"I used recreational mathematics items with my classes for the first time this year; I intend to use them next year even more than I did this."

"They offer a welcome break in class routine, both for the pupils and the teacher."

"This material is invaluable in providing a 'starting point' for recitation in mathematics classes."

"I feel that it has resulted in many of my pupils applying themselves to a better study of basic mathematics."

"I have found that children carry the interest home that was stimulated by recreational mathematics in the classroom; they frequently bring similar material into the classroom from their homes. I feel this is an excellent public relations medium for our school."

"It helps to let pupils know that a mathematics teacher is a human, normal individual."

"Teaching is many times best accomplished by finding things to do that are fun and incorporating these activities into materials to be mastered. Recreational mathematics is such a medium."

"I feel that recreational mathematics is a device that offers teachers, especially beginning teachers, an excellent opportunity to establish the security of acceptance with their classes."

"Making mathematics a pleasure for pupils is not a crime, but in my opinion a goal."

"I believe that recreational mathematics materials stimulate interest and lend a note of encouragement to pupils who dislike dull periods. They are especially helpful with slow groups."

Most of the persons, other than teachers, requesting and receiving the recreational mathematics materials, indicated that they were put into the hands of

regular teachers or student teachers with the suggestions that they be utilized for supplementary classroom material, material for mathematics clubs, and material for teacher workshops. Opinion was expressed that these materials were most useful on the senior high school level and of greatest value to teachers as an aid in establishing good pupil-teacher relations in the classroom.

Many of the persons responding to the questionnaire sent recreational mathematics materials that they found useful in conducting their own classes. These materials included puzzles, games, devices, suggested projects, number oddities, fallacies, stories, problems, books, pamphlets, and bibliographies. A bibliography of selected references, compiled from the material sent to the writer, is included with this article.

The findings of the study are summarized as follows:

1. Recreational mathematics materials are being used by mathematics teachers in the secondary schools in many different ways; including the materials as a part of the regular lesson plan is the way preferred by the greatest number of teachers responding to the questionnaire used in this study.

2. Most of the teachers responding to the questionnaire felt that the use of recreational mathematics materials improved their classes and resulted in stimulating interest for mathematics in most of their pupils.

3. Responses to the questionnaire indicated that recreational mathematics materials were most useful in the ninth- and tenth-grade subjects, including general mathematics, elementary algebra, and plane geometry; such materials were also useful in teaching special classes.

4. Most of the teachers who responded to the questionnaire had tried recreational mathematics materials in their classrooms for a semester or more and were convinced of the usefulness of such materials in improving pupil attitudes and making learning more effective.

Selected Bibliography of Recreational Mathematics Publications

Note: Most of the following selections are on the shelves of the larger libraries and book stores; some are available directly from the authors and publishers.

1. Abbott, E. A. *Flatland.* New York: Dover Publications, 1952. 109 p.
2. Bakst, Aaron. *Mathematics, Its Magic and Mastery.* New York: Van Nostrand Co., 1952. 790 p.
3. Ball, W. W. R. *Mathematical Recreations and Essays.* New York: The Macmillan Co., 1947. 418 p.
4. Brandes, L. G. *Math Can Be Fun.* Alameda, California: The Author, 1727 Santa Clara Ave. 1950. 82 p. $1.00.
5. Brown, J. C. *Easy Number Tricks.* Pelham, New York: Pelham Public Schools, 1936. 53 p.
6. Collins, A. F. *Fun With Figures.* New York: Appleton-Century, Inc., 1928. 253 p.
7. Cuthbert, W. R. *Days for Dates.* Alhambra, California: The author, 1944. 31 p.
8. Dudeney, H. E. *Amusement in Mathematics.* New York: Thomas Nelson and Sons, 1921. 258 p.
9. Degrazia, Joseph. *Math is Fun.* London: George Allen and Unwin, Ltd. 1949. 159 p.
10. Freeman, Mae. *Fun With Figures.* New York: Random House, Inc., 1946. 60 p.
11. Gamow, George. *One, Two, Three . . . Infinity.* New York: The Viking Press, 1949. 340 p.
12. Heath, R. V. *Mathemagic.* New York: Simon-Schuster Company, 1933, 138 p.
13. Hogben, Lancelot. *Mathematics for the Millions.* New York: Norton and Company, Inc., 1951. 697 p.
14. Jones, S. I. *Mathemat ical Nuts.* Nashville, Tenn.: The author, Li e nd Casualty Bldg., 1936. 340 p.
15. Jones, S. I. *Mathematical Wrinkles.* Nashville, Tenn.: The author, Life and Casualty Bldg., 1929. 361 p.
16. Kasner, Edward and Newman, J. R. *Mathematics and the Imagination.* New York: Simon and Schuster, Inc., 1940. 380 p.
17. Kraitchik, Maurice. *Mathematical Recreations.* New York: Norton and Company, Inc., 1947. 418 p.
18. Lee, W. W. *Math Miracles.* Durham, N. C.: The author, Box 105, 1950. 83 p.
19. MacHuisdean, Hamish. *Yesterday's Impossibilities.* Glasgow: Fraser, Edward and Co., 1943. 52 p.
20. MacMahon, P. A. *New Mathematical Pastimes.* New York: The Macmillan Co., 1930. 116 p.
21. Merrill, H. A. *Mathematical Excursions.* Boston: Bruce Humphries, Inc., 1933. 145 p.
22. Mott-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts.* Philadelphia: The Blakiston Company, 1946. 248 p.

(*Continued on page 336*)

The President's Page

Lincoln, Miami Beach, and Kalamazoo

These lines are written just after the page proof for the official program of the Thirty-First Annual Meeting of The National Council of Teachers of Mathematics, held last month in Atlantic City, was returned to the printer. The members of our largest Affiliated Group, the Association of Mathematics Teachers of New Jersey, and the officers of the National Council worked hard to make the 1953 annual meeting the "biggest and best", and more importantly, a meeting of great value to those who attended and to our cause which is that of the improvement of mathematics education at all levels of instruction. Reports on the annual meeting will appear in fall numbers of The Mathematics Teacher.

My thoughts now (February 3) turn back to the 1952 Christmas meeting at Lincoln and ahead to Miami Beach and Kalamazoo. It has been pleasant indeed to receive many fine letters of appreciation of the Lincoln meeting and it is also pleasant to look forward to our summer meetings with the thought that the National Council never stands still but constantly moves forward to new areas and new work.

A picture of a Christmas tree adorned with decorations of mathematical design, prepared in the mathematics classes of Josephine Wagner of Sioux Falls, South Dakota appears on the next page. This tree stood in the lobby of the Hotel Lincoln during the Christmas meeting. A picture of the tree was shown on the front page of one of the daily newspapers of Lincoln along with a headline story of the meeting. In one sense the tree was typical of the efforts of the people of the Lincoln area to provide the best possible local setting for the National Council convention. The hospitality of the Nebraska Branch of The National Council of Teachers of Mathematics was excellent. A most friendly atmosphere prevailed and a real enthusiasm for the National Council was everywhere in evidence. The fine Lincoln experience gives new support to our belief that it is extremely worthwhile to hold meetings in all parts of the country.

The importance of the one day meetings of the National Council, held as part of the annual National Education Association conventions, must not be overlooked. These meetings have been held since the affiliation of the National Council with the N.E.A. in 1950. This summer a one day program is planned for June 29 at Miami Beach, Florida. We are proud of the fact that many mathematics teachers are annually sent as delegates to the N.E.A. convention, and we hope these teachers take advantage of the National Council program in Miami Beach. Other teachers in the southeastern area will not want to miss this opportunity as well as teachers who may happen to be vacationing in Florida. The program is planned around the theme of needs of mathematics students with reference to identification of the needs, meeting the needs, and providing a challenge to students to do their best work.

It had been planned to present on this page in this number a report on curriculum studies in mathematics carried out in the past three years by state groups under the sponsorship of State Departments of Public Instruction. Letters were addressed to the state superintendents in all of the forty-eight states and in Hawaii and Alaska. This plan has not been carried out because of the desire to report again on our meetings and also because it is my understanding that a much more detailed report, than would be possible in this space, is under preparation by another agency. Reference however can appropriately be made to these important publications, since teachers associated with a

number of these publications have been invited to discuss the materials, prepared in their states, at the Summer Meeting of the National Council to be held at the Western Michigan College of Education, in Kalamazoo, August 24–26.

Invitations to take part in the Kalamazoo program were sent to teachers associated with the preparation of the following recent publications:

Florida: *Functional Mathematics in the Secondary Schools*

New York: *A Suggested Outline of Content, Teaching Organization, and Time Schedule for the Seventh, Eighth, and Ninth Grade Programs in Mathematics; also for the Tenth, Eleventh, and Twelfth Grade Programs in Mathematics*

Pennsylvania: *A Course of Study in Mathematics for Secondary Schools*

Wyoming: *Course of Study for Arithmetic, Grades 1–8.*

Invitations were also issued to representatives of groups working on curriculum materials, in Iowa and Minnesota. While it was not possible for all to whom invitations were sent to accept, this list is included with the hope that teachers who are involved in preparation of curriculum materials may be interested in the products of similar activities in other states. Copies of most of these materials will be sent free or for a small charge, upon request to the state department of public instruction by which they were published.

Of course, these discussions of curriculum materials will be only one part of the Kalamazoo program, which can be found on page 386 in this number of THE MATHEMATICS TEACHER. Michigan is an ideal place for a summer meeting and we hope that many teachers will be able to take advantage of the program planned and also a few days (or longer) trip in one of America's finest summer vacation spots.

RESULTS OF 1953 ELECTION

Newly elected officers of The National Council of Teachers of Mathematics were announced at the annual meeting in Atlantic City as follows:

Vice-President (Junior High School)—Mary C. Rogers, Westfield, New Jersey

Vice-President (College)—H. Glenn Ayre, Macomb, Illinois

Board of Directors—

Howard F. Fehr, New York, New York

Phillip S. Jones, Ann Arbor, Michigan

Elizabeth J. Roudebush, Seattle, Washington

1952 Christmas Tree at Lincoln
(Courtesy of Macdonald Studios)

NEW EDITOR APPOINTED

At the meeting on April 8 in Atlantic City, the Board of Directors of The National Council of Teachers of Mathematics appointed Henry Van Engen to the position of editor of THE MATHEMATICS TEACHER for a three-year term beginning

June 15, 1953. Mr. Van Engen is Head of the Department of Mathematics at Iowa State Teachers College, Cedar Falls, Iowa.

Other Appointments

Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana was elected by the Board of Directors to serve on the Board of Directors for the unexpired term of the new editor of The Mathematics Teacher, who has resigned from the Board, effective June 15. Mr. Van Engen was elected to the Board in 1951 for a three year term.

The Board of Directors has appointed Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland, to the position of Recording Secretary of the National Council for the one-year term beginning at the close of the Thirty-First Annual Meeting in Atlantic City.

John R. Mayor
President

Nominations for the Board of Directors

The By-Laws of The National Council of Teachers of Mathematics require that "the Nominations and Elections Committee shall cause an announcement to be published in the official journal, at least five months before the annual meeting, inviting members of the Council to suggest nominees for elective offices."

The 1953 ballot will present candidates for the following offices:

President (to serve for two years)
A Vice-President representing senior high school mathematics (to serve for two years)
A Vice-President representing elementary school mathematics (to serve for two years)
Three members of the Board of Directors (to serve for three years).

The candidates for Directors may represent any level of instruction. The present geographic plan suggests that only one of the nine Directors may come from one state. Present Directors who will hold office beyond April, 1954 are from the states of Michigan, New York, Ohio, Texas, Virginia, and Washington. This limitation has usually not been applied to other officers.

The Nominations and Elections Committee invites members of the Council to suggest persons to be nominated and to submit brief statements of their qualifications and their participation in local and national affairs pertaining to mathematics. Suggestions should reach the committee as soon as possible, but not later than September 1, 1953. Suggestions may be sent to any member of the Nominations and Elections Committee: Mary A. Potter, Chairman, City Hall, Racine, Wisconsin; Ida May Bernhard, Texas Education Agency, Austin; Harry W. Charlesworth, East High School, Denver, Colorado; Martha Hildebrandt, Proviso Township High School, Maywood, Illinois; Lesta Hoel, Mathematics Supervisor, Portland, Oregon; Virgil Mallory, State Teachers College, Montclair, New Jersey; W. W. Rankin, Duke University, Durham, North Carolina; Rolland R. Smith, Mathematics Supervisor, Springfield, Massachusetts; Ben A. Sueltz, State Teachers College, Cortland, New York; Henry Van Engen, Iowa State Teachers College, Cedar Falls.

History of the Association of Teachers of Mathematics in New England, 1903-1953*

By William R. Ransom
Tufts College, Medford, Massachusetts

The Association of Teachers of Mathematics in New England was first called together by the superintendent of schools in Boston, E. P. Seaver, in 1903. A committee consisting of professors from Harvard University, Wellesley College and the Massachusetts Institute of Technology, and five teachers in secondary schools announced a meeting to be held in the Boston Latin School on April 18, 1903. By a singular chance that date came again on a Saturday fifty years later, when the semicentennial of the Association was celebrated by a dinner, a history, and a festspiel.

From the first this group has been remarkable for the hearty cooperation of the people from colleges and preparatory schools. Its interests have been wide, extending from the teaching of arithmetic to recent developments in higher mathematics.

At the end of the first year the Association published a booklet which in eight pages described the achievements of the year. At the end of ten years another booklet was published devoting forty-three pages to general questions about the improvement of mathematical teaching. At the end of twenty years, a committee, of which only Mr. Hobbes is now living, gave in ten pages[1] a detailed account of the activities of the Association to that time. These reports are all preserved in the great Scrap Book kept by the secretaries, and deposited in the safe at Boston University.

The Association has held all day meetings on Saturdays twice a year in Boston and once a year at least 40 miles away, Springfield and Worcester oftenest, and Providence, Hartford, Portland, Andover, New Haven, Concord, New Hampshire, Middletown, Manchester, New Hampshire, Mt. Holyoke, Northampton, Storrs, Durham and over a dozen schools in the suburbs of Boston. There have been joint meetings, once in New York City with the Association of Middle States and Maryland, several times with the New England Association of Schools and Colleges, and with the National Council of Teachers of Mathematics.

The Association is particularly indebted to Boston University which has been its generous host on more than eighty occasions, at more meetings than all other schools and colleges put together.

The purposes for which the Association was formed were stated by those who called it together. These purposes were to increase the importance of the teaching of mathematics in the schools, to make it more effective, and to stimulate the teachers. The early programs were usually concerned with general considerations. However on December 8, 1906, the program introduced the Harvard entrance examinations, and for some years the largest gatherings occurred when something about entrance examinations appeared on the program. After a while the college board examinations excited the same interest. A committee of the Association prepared some "pattern examinations" which were circulated among the members and denounced as warmly as the entrance examinations for whose reform they were intended. The Association sponsored committees on the teaching of arithmetic and geometry, a great deal of work

* Presented at the Semi-Centennial meeting of the ATMNE, Saturday, April 18, 1953.

[1] "Twenty Years of the Association of Teachers of Mathematics in New England" The Mathematics Teacher, XVII (February, 1924), 94–103.

was done on a syllabus project and a sixteen page booklet on loci was published.

In the first quarter century there were many provisions for discussions, and volunteer contributions from the floor. There were three types of papers prevailing: 1. General considerations affecting the content of courses and their administration, 2. Expositions as to "How I teach . . . ," and 3. Expositions of interesting mathematical topics outside the range of many teachers' experience. Under 2, we note:

Factoring, problems, reversal equations, prismatoidal formula, equations to beginners, stripes, ratio and proportion, mathematics clubs, quadratics, locus problems, rounding off, slide rule, radicals, graphs, squared paper, calculus in high school, and numerical equations.

Under 3, we find the following:

Simpson's rule, two-edged ruler, Einstein's theory (Yes!), history of mathematics, linkages, "non-Euclidean" algebras, parallel postulate, inscriptible polygons, rockets, varieties in space, dynamic symmetry, photogrammetry, metric system, proportional representation, maps, and, correlation.

Speakers who appeared three or more times on the programs of the first twenty-five years included: Barber, Birkhoff, Bouton, Carpenter, Coolidge, Downey, Evans, Farnsworth, Francis, Hapgood, Hawkes, Huntington, Marsh, C. H. Mergendahl, Meserve, Morely, Olds, Packard, Ransom, D. E. Smith, P. F. Smith, Snell, Tyler, Vosburg, Wheeler, and J. W. Young.

Some one started the series of dinners that have become such an important supplement to our all day meetings. The great Scrap Book contains the announcement of a dinner on October 23, 1920, but this was certainly not the first. Ransom's diary shows that he went to thirteen such dinners before this one, the earliest being October 29, 1910. But it is quite certain that they began earlier than 1910. At first they were gotten up by a small clique of college and high school *men* in and around Boston. For several years these dinner meetings were devoted to reviews of new textbooks. Tyler made the arrangements about the program for several years, styling himself "Committee on Literature and Dogma." Later he transferred this job to Ransom who drafted Dow and MacGregor as partners in the enterprise. What happened to this arrangement when he became president in 1919 no one seems to remember, but ultimately the dinners became the charge of the Council that controlled the other activities of the Association. The small group of men who started the dinner series got help from the secretary for the mailing of notices, and by some inadvertence a notice was sent to the whole membership of the Association. Dr. Lennie Copeland of Wellesley College sent in word that she would be on hand—to the consternation of the men in charge of the dinner. There was resistance on both sides: Dr. Copeland insisted on coming—it was a meeting of the Association she argued and she was a member in good standing. The reason the objectors gave for not wanting any women sounds very strange today—*the men wanted to smoke*. But Dr. Copeland prevailed and the women have been welcomed ever since. The book reviewing at the dinners probably ceased before the Twenties. We have no record to inform us, but there was a book review by Dr. Bouton in February of 1919. The dinners have become an exceedingly valuable feature of the Association's work, not only because of the high quality of their programs, but because they greatly increase the intimacy and friendship among the members.

We have lost a great many of those who made the first 25 years successful. We know of two survivors who attended the first meeting. One of them is writing this résumé. A list, similar to the other, of speakers who have appeared repeatedly at our meetings during the second twenty-

five years should include: Adkins, Beatley, Bennett, Brower, Brown, Bruce, Cheney, Coolidge, Copeland, Evans, Farnsworth, Graustein, Haskins, Kee, Leib, C. H. Mergendahl, Mode, Morley, Ransom, Rice, Sayward, Sedgwick, Selleck, R. R. Smith, Spear, Struik, Syer, and Wheeler.

The majority of the speakers have dealt with matters of general interest in regard to programs of studies, reorganization of curricula, purpose and failures in our teaching, use and abuse of tests, methods of stimulating teachers and pupils. There have been a great number of papers of a refreshing and informative sort dealing with such mathematical topics as:

Factoring large numbers
Imaginaries
Trisection
Projective geometry
Skew squares
The complex plane
Simultaneous quadratics
Mathematics clubs
Parallel ruler constructions
Computing machines
Maxima and minima
Road curves and safety
Repeating decimals
Calendar
Financial applications
Ptolemy's theorem
Harmonics
Consumer credit
Euclid and number theory
Order and linear dependence
Electronic navigation
Integral sided right triangles
Mathematical theory of harmony and melody
Statistics
Non-Euclidean geometry
Diophantine analysis
Ratio
Algebraic and transcendental numbers
Perspective triangles
Infinity with and without postulates

The second twenty-five years follows much the same lines that have been indicated, except that the type of presentation of "How I Teach . . . " and the discussions from the floor seem to have passed out. As before the secretary has been the most important officer. The first one, Dodge, of the Roxbury Latin School, had to give up on account of ill health after the first year. Thereupon Evans of the Charlestown High School served five years, followed by Gaylord, of blessed memory, who served over a quarter of a century, leaving us only when he removed to California in 1936. Then came Garland 1936–1945; he earlier served as treasurer and later as president. Bridgess served from 1946–1948 followed by Miss Margaret Cochran, 1948 to the present, and long may she wave—in her efficient handling of the Institutes' finances. She has probably handled more money for the Association than all her predecessors put together.

Lists of members were published in 1924 and in 1931. Since the Institutes began we have had lists of those attending, but have printed no complete list of our membership, which has now reached an all-time high of about four hundred.

The Association has a Connecticut Valley Section. This was started by H. B. Marsh of Springfield the year after he left the presidency of the Association. On March 13, 1920 the mid-winter meeting was held in Springfield and on the agenda for the afternoon was the item "Organization of the Connecticut Valley Section of the Teachers of Mathematics in New England." Its first annual meeting was held that fall in Hartford, November 6, 1920. There was a preliminary organization calling itself an independent society, but the affiliation has been satisfactory and fruitful.

On January 24, 1948, an exceedingly stormy night, Ricci, Miss Collins and Miss Height reported on the Institute they had attended at Duke University. The Association appointed a committee to see about providing a similar institution for New England, and four very successful institutes have been held here at the end of August as follows:

1949 at Wellesley College, directed by Dr. Henry Syer
1950 at Tufts College, directed by Miss Janet Height
1951 at Connecticut College for Wom-

en, directed by C. H. Mergendahl 1952 at Phillips Exeter Academy, directed by J. B. Adkins.

The fifth Institute will be held at Colby College this year under the direction of Miss Ruth Eddy.

The general plan of the Institutes has been to spend a week on a school campus, with lectures at 9 A.M. and 8 P.M. by nationally eminent speakers on subjects important to teachers of mathematics, followed by three periods, in which groups were gathered under competent leaders, devoted to discussions, instruction, and interchange of experiences. Much time was given to laboratory work, planned trips and recreation. Good eating in inspirational companionship loomed large in the attractiveness of the Institutes. Attendance came from twenty-eight different states. There were exhibits of students' work, of teachers' ingenuity, of publications, of games and puzzles.

Each Institute had a picnic and a banquet with mathematical decorations, and there were evening snacks and magic shows and photographs. Trips took the teachers to computation laboratories at Harvard University and Massachusetts Institute of Technology, to the weather bureau, the Weston Seismograph, the Planetarium and Museum of Science, to the Marine Museum at Mystic, the Coast Guard Academy at New London, and to the Towle Silver Company. There were trips by bus along the ocean and to the White Mountains.

The leaders of the Institutes have had very efficient aid from numerous and very active committees, and the membership of the association has been very cooperative and very well repaid by the notable success of these affairs.

Recreational Mathematics

(*Continued from page 329*)

23. *Multi-Sensory Aids in the Teaching of Mathematics.* National Council of Teachers of Mathematics, Eighteenth Yearbook. New York: Bureau of Publications, Teachers College, Columbia University, 1945. 455 p.
24. NORTHROP, E. P. *Riddles in Mathematics.* New York: Van Nostrand Co., 1944. 262 p.
25. POLLOCK, SAUL. *Mathematical Paradoxes and Recreations.* Los Angeles: American Association for the Advancement of Visual Instruction in Mathematics, 2512 S. Vermont Ave., 1937. 108 p.
26. RULON, P. J. *Brain Teasers.* Boston: Page and Company, 1932. 250 p.
27. STEINHAUS, HUGO. *Mathematical Snapshots.* New York: Oxford University Press, 1950. 266 p.
28. *Story of Measurement* (folder). Dearborn, Michigan: Ford Motor Company, (Sent upon request.)
29. THOMPSON, J. E. and SLOANE, T. O. *Speed and Fun with Figures.* New York: Van Nostrand Co., 1922. 559 p.
30. TRAVERS, JAMES. *Puzzling Posers.* London: George Allen and Unwin, Ltd., 1952. 80 p.
31. *The Amazing Story of Measurement.* (Illustrated pamphlet). Saginaw, Michigan: Lufkin Rule Company, 1949. 24 p. (Sent upon request.)
32. *The Story of Figures.* Detroit, Michigan: Burroughs Adding Machine Company, 1928. 56 p. (Sent upon request.)

ATTENDANCE RECORD OF THE THIRTEENTH CHRISTMAS MEETING

Hotel Lincoln, Lincoln, Nebraska
December 29, 30, 31, 1952

State	*No.*	*State*	*No.*	*State*	*No.*
Arkansas	5	Maryland	2	North Dakota	1
California	2	Michigan	1	Oklahoma	3
Canal Zone	1	Minnesota	6	Ohio	4
Colorado	9	Mississippi	1	Pennsylvania	1
District of Columbia	3	Missouri	5	South Dakota	3
Illinois	18	Montana	2	Tennessee	1
Indiana	4	Nebraska	194	Texas	5
Iowa	12	New Jersey	1	Wisconsin	8
Kansas	32	New Mexico	2	Wyoming	1
Louisiana	1	New York	2	Total	330

Annual N.C.T.M. Summer Meeting with N.E.A.

Miami Beach, Florida, Monday, June 29, 1953

9:00 A.M. **Registration**—Shelborne Hotel, Card Room

MORNING SESSION

9:30–11:45—Shelborne Hotel, Card Room

Introduction of Officers of N.C.T.M.

Greetings from Our National Council President

Theme: *Challenging Students in Mathematics Classes*

Presiding: Kenneth P. Kidd, University of Florida, Gainesville, Florida

Practical Mathematics Challenges the Student

F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

Challenging the Gifted Student

VERYL SCHULT, Public Schools, Washington, D. C.

Devices and Techniques Which Arouse Interest

AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

Discussion

12:15 P.M. **Luncheon**—Shelborne Hotel, South Room

(Luncheon, $2.75. Send reservations and check to Miss Florence Shaffer, 631 N. E. 138th Street, North Miami, Florida before June 22, 1953.)

AFTERNOON SESSION

1:45–3:00 Panel Discussion: *What Are the Needs of Students in Mathematics?*

Moderator: HOWARD GALLANT, President, Florida Council of Teachers of Mathematics, Hillsborough High School, Tampa, Florida

Mathematics Needed by the Consumer

F. G. LANKFORD, JR., University of Virginia, Charlottesville, Virginia

Mathematics Needed in Preparation for College

F. W. KOKOMOOR, University of Florida, Gainesville, Florida

Mathematics Needed by the Engineer

HARVEY F. PIERCE, Secretary-Treasurer, Maurice H. Connell and Associates, Consulting Engineers, Miami, Florida

Mathematics Needed by the Scientist

PHILIP W. CARTER, Assistant Professor of Physics, University of Miami, Coral Gables, Florida

Mathematics Needed in Aeronautics

W. W. MOHLHENRICH Special Assistant Industrial and Personnel Relationships, Eastern Airlines, Miami, Florida

3:00–4:00 P.M. Theme: *How Can These Needs be Met?*

Presiding: BERNHARD P. REINSCH, Florida Southern College, Lakeland, Florida

Making Mathematics Live

CARL N. SHUSTER, State Teachers College, Trenton, New Jersey

Using the Functional Approach in Teaching

WILLIAM A. GAGER, University of Florida, Gainesville, Florida

Discussion

General Chairman

William A. Gager, University of Florida, Gainesville, Florida

Program Chairman

Charlotte Carlton, Miami Beach High School, Miami Beach, Florida

Luncheon Plans

Dade County Council of Teachers of Mathematics, Florence Shaffer, President

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

Anyone who has a learning device which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. Or if that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

MAKING MODELS WITH CHALK BOXES

If there is any one thing that is readily available to all teachers at no cost, it is empty chalk boxes. Such boxes may be used as frames for various kinds of string models. Those shown in the picture represent three types of intersections (Fig. 1). The model at the top is an illustration of the intersection of two cylinders. The one at the lower left illustrates the intersection of a cylinder and a cone, and the one at the lower right shows the various conic sections.

Using chalk boxes as models may not appear to be a particularly novel idea at first sight, but a closer look at the "box" will reveal that this type of box is quite suitable for the uses described above. Because the edges of the box are either dovetailed or grooved the box has the advantage of being quite durable, and in some cases the string used for a particular model may actually add to the rigidity of the box. Also, since all such boxes are uniform in size and shape the problem of storing them

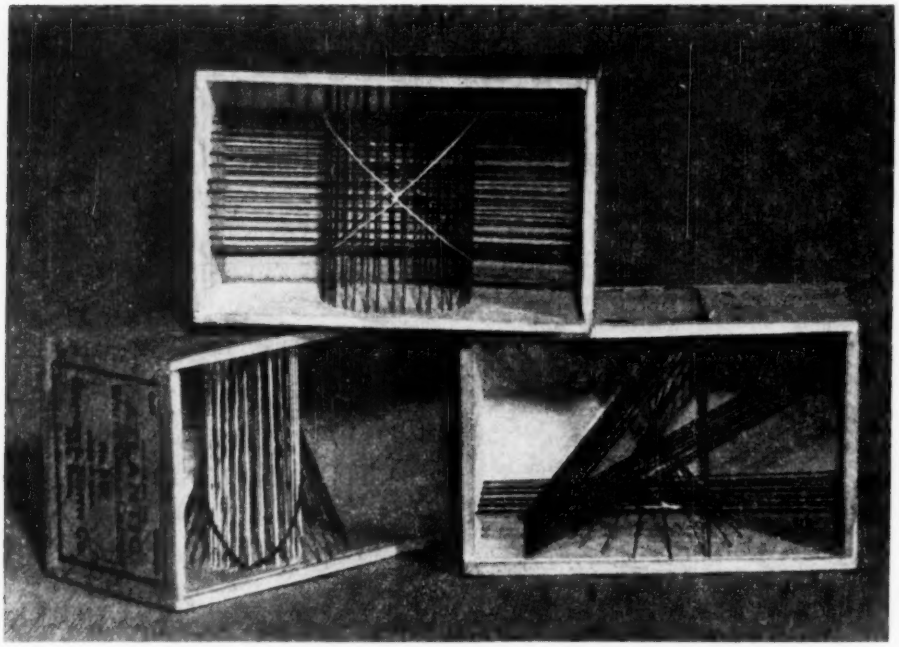

FIG. 1

is not nearly as difficult as it might be. If the school custodian can be persuaded to save the box covers these can be used to help keep the string models clean from year to year. Anyone who has ever tried to dust a string model will undoubtedly welcome this tidy little suggestion for avoiding a nasty task.

In addition to their use in the construction of models as described above, chalk boxes, when adequately labeled, also provide a handy means of storing the many small models that are used in the mathematics classroom.

FRANK HAWTHORNE
Hofstra College
Hempstead, New York

A THREE DIMENSIONAL GRAPHING DEVICE

The graph of an equation in two variables can usually be displayed on a sheet of graph paper. Its locus will in general be some kind of plane curve or straight line.

Graphing an equation in three variables is not quite so simple, however. Usually this is a problem involving the representation of some kind of surface (or a line in space). In practice this is often accomplished by making models from plaster casts or wood, but such a plan has its drawbacks. For one thing the materials in the finished model cannot be re-used to represent other equations, and also, if a poor choice has been made for the unit of representation no modifications can be made later on.

The three dimensional graphing device suggested in this article has been designed to eliminate such shortcomings. It consists of 144 movable wooden rods which can be adjusted to locate points that satisfy the equation of some surface or show the relationship between other kinds of related data. From the photograph in Figure 2 it can be seen that one end of each rod (A) is square, and that all 144 fit into a square wooden box (B), which in turn fits into a square wooden base frame (C). Each wooden rod is a single unit but is made from two separate pieces—a wooden block 3″ long with a square end $\frac{1}{2}'' \times \frac{1}{2}''$, and a 1/8″ round dowel 5″ long. To fasten the dowel to the block drill a hole 1″ deep in one end of the block and force the dowel into it. The diameter of this hole should be just large enough to give a tight fit. The overall length of a completed rod should be 7″.

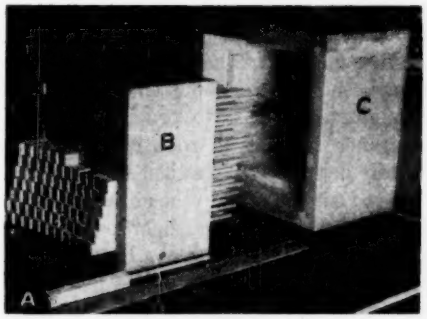

FIG. 2

The inside dimensions of the box should be approximately 6″×6″ and 3″ deep. A fact which is not altogether obvious from an examination of the picture is that only the "block" ends of the rods fit into this box. Holes must be drilled through the bottom of it to allow each of the 144 dowels to protrude as the picture indicates. Note that the "block" ends of the rods are arranged in the box in a 12×12 pattern. Note also that the inside dimensions given above are labelled "approximate". Actually only the 6″×6″ dimensions (length and width) may be different when the reader builds his device. The only helpful thing that can be stated about these two dimensions is that their magnitude must be such that when any one rod is moved up or down it must stay in place simply from the support it gets from the surrounding rods or the box itself. The rods are not fastened to each other or to the box in any way whatsoever, except that the dowel at the end of each fits through a hole in the bottom as already explained. A rod may be adjusted simply by pushing or pulling on its attached

dowel. All this means that the box may not be so small as to render the rods immovable, nor yet so large as to leave any one of them without support when adjusted.

The wooden base frame into which the box fits should be open at both ends and have a $\frac{1}{2}'' \times \frac{1}{2}''$ molding nailed around the inside near one end (about $1\frac{1}{2}''$ from the opening). The purpose of the molding is to support the box. In this way the box (and its rods) can be supported in a vertical position. The depth of the base frame should be about 6″, but deep enough in any case to provide sufficient clearance so that all rods may be completely depressed. To adjust the height of any rod simply pick up the entire device—frame and all—and push or pull on the proper dowel. Each rod can be set at any height up to $3\frac{1}{2}''$. As an added feature all rods may be marked according to some convenient scale if desired.

This device has a number of rather interesting uses. As previously mentioned it may be used to "graph" the different quadric surfaces (those defined by second degree equations in three variables), provided of course, that the origin is appropriately chosen. The positions of the x and y-axes painted on the device would seem to imply that the origin must be taken as the corner toward the observer (with the positive z-axis extending upward), but this is certainly arbitrary. The markings on the device simply indicate one possible position for the origin. When this point is chosen to represent the origin then the device is simply the "first octant" of a three dimensional reference frame. Consider, for example, Figure 3. It is a picture of a portion of a paraboloid of revolution which has an equation of the form $(x^2/a^2)+(y^2/a^2)=cz$, but only the part which belongs to the first octant is shown.

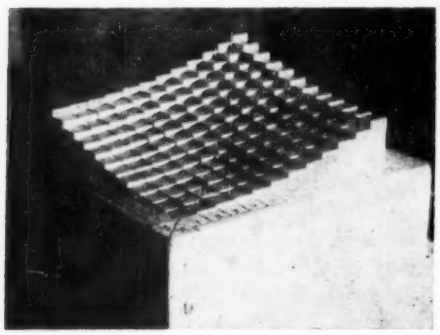

FIG. 3

Figure 4 pictures the graph of a surface with the origin removed to a point about 2″ above the center of the top of the box.

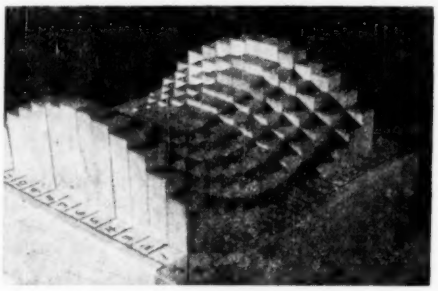

FIG. 4

The surface represented is a hyperbolic paraboloid which has an equation of the form $(x^2/a^2)-(y^2/b^2)=cz$. All eight octants can be accounted for in this illustration.

The visualization of surfaces defined by equations in three variables is not the only use that can be made of this device. It is also useful for comparing other types of related data such as heights, weights, and oxygen requirements of a population sample. The reader will note that since the device makes use of three axes of reference the classes of data must be confined to three kinds.

It is of interest that this device can be used both inductively and deductively. When used in inductive fashion the available data is plotted in the hope of obtaining a general relationship. The representation of this relationship may turn out to be some kind of surface or it may be a stereograph.[1] The possibility of visualizing all

(*Continued on page 345*)

[1] "Devices for a Mathematics Laboratory," THE MATHEMATICS TEACHER; XLV (February, 1952), 106–8.

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

81. The Pentagon and Betsy Ross

Many years ago I read an anecdote in which Betsy Ross was supposed to have objected to George Washington to six pointed stars in the proposed new American flag because of some connection with British insignia. When given the explanation that six pointed stars were easier to make, she replied by folding a piece of cloth or paper and with but one snip of her scissors cutting out a perfect five pointed star.

Perhaps some reader can document this anecdote, I have never been able to do so, but two recent publications recall it. The first was an advertisement published by the Wrigley Company on page 605 of the December, 1952, *N.E.A. Journal.* It included the following directions for "A Five Pointed Star with One Snip" with an acknowledgement to the *Popular Mechanics Christmas Handbook of Ideas, Toys, Gifts You Can Make.* Essentially the same instructions appear on page 56 of *Fun with Figures* by Mae and Ira Freeman (New York: Random House, 1946).

Starting with any paper with $8\frac{1}{2}\times 11$ proportions fold it in two with the center crease, POQ, in the $8\frac{1}{2}$ direction to give a new rectangle $8\frac{1}{2}$ by $5\frac{1}{2}$. The upper end of the crease is then folded to the center, Q', of the bottom $5\frac{1}{2}$ side as shown in Figure 1. The lower end P, of the original crease is then folded up over the top part of the original crease as shown in Figure 2. Finally the crease OS is folded over onto the crease OQ' as in Figure 3 and the whole is cut along BA. Angle OAB determines whether a conventional five pointed star, a pentagon, or a decagon is obtained. Betsy Ross no doubt found that when she made the angle OBA equal to one half of angle BOA that the regular five-pointed star resulted.

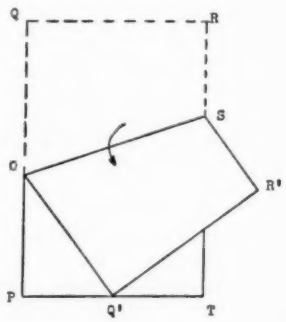

FIG. 1

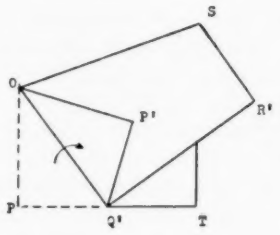

FIG. 2

This sort of thing is of most interest and value when the relationship to mathematics is clearly pointed out and exploited. In this case the angle POQ' is determined

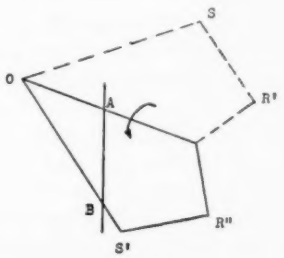

FIG. 3

by the proportions of the paper and in turn determines one of the central angles between lines from the center to successive points of the star. The other angles will closely approximate POQ'.

$$\text{Tan } POQ' = \frac{PQ'}{OP} = \frac{2.75}{OP}.$$

Further, $PO^2 = OQ'^2 - 2.75^2$ while $OP + OQ' = 8.5$. From this, angle $POQ' = 35°\ 51'\ 20''$. The corresponding angle for a perfect star would be 36°.

Additional questions could be raised concerning the sizes of other central angles and the possible relationships between the dimensions of standard letter size paper and the "golden rectangle," both of which we now see to be related to the pentagon-decagon problem. Perhaps someone would like to explore this for a later note in this department.

The second recent printed discussion of cutting a five pointed star is found on page 116 of *Things to Make and Do* by Esther M. Bjoland (Chicago: Standard Education Society, 1952). Here is a brief reference again to the Betsy Ross-George Washington story, but with no documentation. Miss Bjoland's construction, however, begins with a *square* piece of paper instead of $8\frac{1}{2} \times 11$, but then for her second fold she brings point Q of our Figure 1 to a point $\frac{2}{3}$ of the way from P to T rather than $\frac{1}{2}$ way as we did.

Perhaps some reader or student would like to figure out the mathematical justification of the second and the relative accuracy of the two methods?

82. Trisection!

In *Miscellanea 4, 20, 47* [THE MATHEMATICS TEACHER. Vol. XLIII (October, 1950) pp. 278–9; Vol. XLIV (March, 1951) pp. 194–5; Vol. XLV (March, 1952) pp. 234–5] we presented notes on trisection in spite of our declared intention of avoiding the topic. We did this under the conviction that the topic does have teaching values and that something about each presentation was sufficiently new or different to warrant publication of the note.

Here we present three approximate constructions which may be regarded as supplements to Chapter IV of Robert C. Yates' *The Trisection Problem* (1942) which is, I believe, still available from the author at the United States Military Academy, West Point, New York. The first of these notes analyzes the degree of approximation and simplicity of the construction and compares it in these respects with constructions discussed by Yates. Perhaps some reader or student might care to do this for the other two approximations below and even to extend the chart on page 56 of Yates' book to include these methods and perhaps angles beyond 90°.

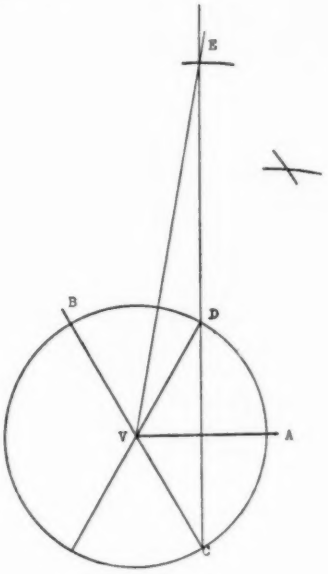

FIG. 4

AN INTERESTING TRISECTION APPROXIMATION

Mr. C. R. Lindberg of Saratoga, California recently proposed an approximation to the trisection of the angle. His method with one slight refinement follows.

In Figure 4, using V, the vertex of

the angle, as a center, let a circle intersect the sides of the angle at A and B. Extend BV to C on the circle. Find D the mid-point of $\overset{\frown}{AB}$. Draw CD and extend to E so that $DE=BC$. Draw EV. Then $\angle BVE$ is approximately $1/3\angle BVA$.

The error in Lindberg's approximation is easily seen to be $\angle EVD-1/6\ \angle BVA$. In $\triangle EVD$, $\angle E+\angle V=\angle VDC=1/4\angle BVA$; and sin $V=2$ sin E. The error can be found by merely consulting a table of sines. For example, if $\angle BVA=60°$, the error is less than $1'$, for sin $10°<2$ sin $5°$, and sin $10°\ 1'>2$ sin $4°\ 59'$. By solving the triangles

$$\tan \angle EVD=\frac{2\sin\frac{\angle BVA}{4}}{1+2\cos\frac{\angle BVA}{4}}.$$

The errors for 90°, 120°, and 180° are respectively $3'$, $6'$, and $22'$ correct to the nearest minute.

Yates has compared five approximations and their errors, but for only one of these, Durer's, are errors tabulated

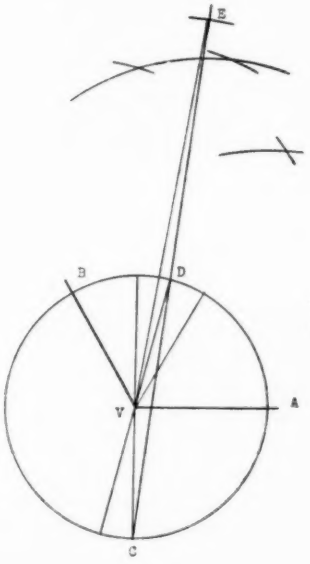

FIG. 5

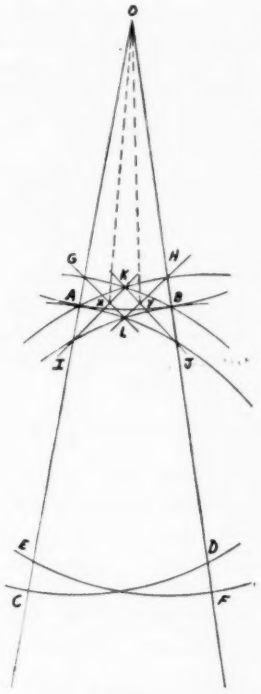

FIG. 6

for angles greater than 90°. Durer's maximum error is more than $31'$ compared with Lindberg's $22'$. Lindberg's method requiring only 4 arcs, 3 lines, the extension of one side of the angle, and carrying one segment, is simpler than any but D'Ocagne's which requires 4 arcs, 3 lines, and the extension of one side of the angle. However, D'Ocagne's method permits errors of $21'$ for some acute angles compared with a maximum of $6'$ by Lindberg's method. Moreover, Lindberg actually divides the angle and his method is simpler if the angle is to be divided into three parts.

The most accurate method for acute angles mentioned by Yates is that of Kopf-Perron which permits errors of only $15''$ to the nearest second. An extension of Lindberg's method, requiring 6 arcs, 5 lines, and carrying one segment, is simpler than that of Kopf-Perron and permits errors of not more

than 3″ for any acute angle and not more than 20″ for any angle less than 180°. In the extended method (Fig. 5) D is located so that $\overset{\frown}{BD} = (3/8)\overset{\frown}{BA}$, $\angle VDC = 1/16 \angle BVA$, and the error is $\angle EVD - (1/24) \angle BVA$.

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The second approximation (Fig. 6) is the work of Edward T. Richardson, Springfield and Plymouth Roads, Springfield, Pennsylvania. The method of construction is as follows:

1. Use any convenient radius and draw arc $\overset{\frown}{AB}$.
2. With A and B as centers and the same radius draw arcs $\overset{\frown}{EF}$ and $\overset{\frown}{CD}$.
3. Use the same radius and the points C, D, E and F as centers to draw the arcs $\overset{\frown}{IB}$, $\overset{\frown}{AH}$, $\overset{\frown}{AJ}$, and $\overset{\frown}{GB}$.
4. Draw the lines KI, KJ, LG, and KH. (K and L are the intersections of the arcs.)
5. X, the intersection of lines KI and GL and Y, the intersection of lines HL and KJ are the points of (approximate) trisection.

Mr. Richardson states, "The angle must be less than 30 degrees. If greater than 30 degrees, then bisect the angle enough times to secure an angle less than 30 degrees."

The third approximation (Fig. 7) is the work of John Shields, 44 Brook Street, Corry, Pennsylvania, a student of Felix M. Matthews, 328 Concord Street, who sent it in for his pupil. The construction is as follows:

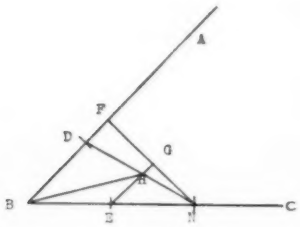

Fig. 7

(1) With any fixed radius and B and E as centers, locate points D, E, N, such that $BD = BE = EN$.
(2) Draw DN.
(3) From N construct NF, perpendicular to AB at F.
(4) From E construct EG perpendicular to NF at G.
(5) The intersection of EG and ND determines H. BH is approximately a trisecting line, i.e., $\angle HBC \approx 1/3 \angle ABC$.

83. The Problem of Napoleon

One of the interesting problems concerning constructions related to the circle is "Given a circle, to construct the position of its center and the length of its radius." This problem has received the attention of many famous mathematicians such as Lagrange, Laplace, and Lemoine. However, Lanascol says it was given the name "Napoleon Problem" by German

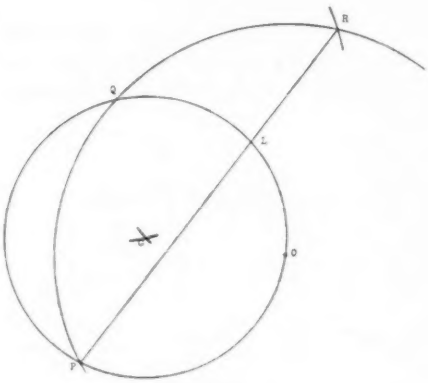

Fig. 8

mathematicians.[1] Lanascol gives the constructions of Mascheroni and Herbst, using only the compasses.

The following construction, credited to Swale (1830), is found in Mackay's *Plane Geometry* and *Harper's Euclid* by Langley and Phillips (1906, p. 199). Mackay says this construction probably is the simplest

[1] A. Quemper De Lanascol, *Géométrie du Compas*. (Paris: 1925), p. ix. See also page 65 ff. "The Napoleon Problem" is solved on page 72.

known. The steps in the construction are as stated in the *Harper's Euclid*.

With any point O (Fig. 8) on the given circle as a center, and any radius, describe an arc intersecting the given circle at points P and Q. With Q as a center, and the same radius, describe an arc intersecting the forementioned arc at point R. Draw the line PR, intersecting the given circle at point L. Then LR is the length of the required radius. The center of the given circle may be found by means of two interesecting arcs.

This construction can be verified by analytic methods, and the plane geometry proof is challenging.

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Editor's Note: There are several interesting relationships between mathematics, Napoleon Bonaparte, and the geometry of the compasses alone.

This latter geometry was first expounded by Georg Mohr, a Dane, in his *Euclides Danicus* published in two editions, one Flemish, one Danish, in Amsterdam in 1672. This work remained unknown and unappreciated until "rediscovered" and republished with a German translation in 1928 by the Danish mathematician Johannes Hjelmslev. In the meantime the Italian Lorenzo Mascheroni published his independently developed *Geometria del Compasso* in Pavia in 1797, and as a result this geometry is often called *Mascheroni geometry*.

Henri Lesbegue in his posthumously published *Leçons sur les Constructions Géométriques* (Paris: 1950) summarizes these facts and repeats from the French translation by Carette of Mascheroni's work (Paris: 1798) an interesting story. Napoleon, on December 11, 1797, the day after the ceremony in Paris celebrating the peace of Campo Formio which had concluded his Italian campaign, met mathematicians Laplace and Lagrange at a social function. He thereupon took considerable pleasure in expounding to them the solution, which supposedly he had picked up in Italy, of several elementary geometry problems using compasses alone. Laplace is said to have replied, "Nous attendions tout de vous, Général, sauf des leçons de géométrie."

E. T. Bell, in his *Men of Mathematics*, reports another exchange between Napoleon, Laplace and Lagrange. Napoleon remonstrated to Laplace that he had not once in his *Mechanique Céleste* mentioned the Author of the universe. Laplace replied, "Sire, I have no need of that hypothesis." Lagrange on hearing of this later remarked, "Ah, but that is a fine hypothesis. It explains so many things." More of Napoleon and mathematics will be found in Bell's Chapter 12, "Friends of an Emperor—Monge and Fourier."

In his *Geometrical Tools* (St. Louis: Educational Publishers, 1947), R. C. Yates refers to Mascheroni as a protégé of Napoleon's. However this may be, the French translation of Mascheroni's book was prefaced with a dedicatory ode "A Boneparte L'Italico," the first few lines of which are quoted by Quemper De Lanascol in the "Avant-Propos au Lecteur" of the book cited by Mr. Mills. I believe that I have also read that Napoleon on occasion challenged listeners to construct a square with compasses alone. Perhaps someone can document this fact. The problem, too, though simple, is challenging, especially for secondary school students.

For a semi-popular account of Mascheroni's geometry and another solution to "The Napoleon Problem," see Richard Courant and Herbert Robbins's *What is Mathematics?* (New York: Oxford University Press, 1941), p. 145 ff.

P.S.J.

Devices

(*Continued from page 340*)

the known data at one time will (in some situations) point the way to the discovery of the general relationship being sought.

The deductive use of the device has already been illustrated. Demonstrating that the graph of a given equation is the surface defined by the locus may be thought of as a deductive process.

One more use seems worth mentioning. The device can also be used to illustrate the usual geometric interpretation of double and triple integrals as the volume under a surface.

Finally, it should be stated that the capacity of this device for representing equations and other data is, naturally enought, tied to the number and length of the wooden rods it contains. Should one desire a device which would make it possible to consider a greater range of values for the variables involved than this particular device permits, the directions given above will serve as a useful guide in building a larger model.

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RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
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The Question: Do teachers of arithmetic really understand the processes and concepts of arithmetic?

The Study: Orleans, Jacob S. *The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic.* Publication Number 12. Office of Research and Evaluation, Division of Teacher Education, College of the City of New York. 1952.

"The purpose of the study . . . was to ascertain the extent to which teachers and prospective teachers of arithmetic understand the processes and concepts which are represented by the short cuts which they teach." Dr. Orleans considered as reasonable the assumption "that if the teachers do not themselves understand underlying processes and concepts they cannot get pupils to learn with understanding."

The first step was to construct a free-answer test consisting of 21 questions. In the sample question 234×56 was worked out in the traditional way. Next to the solution was the question, "Why is the second partial product (1170) moved over one place to the left?" The other items were designed to test an understanding of the meaning of partial products in division, division as a short-cut for subtraction, remainders in long division and division of fractions, fractions as quantities vs. ratios, changing fractions to lower or higher terms, inverting in division of fractions, moving the decimal point in division of decimals, per cent and decimal fraction and the base of our number system, among others.

This test was given in the spring of 1950 to 722 persons among whom were 193 undergraduates taking a course in methods of teaching in the elementary school, including the teaching of arithmetic, 340 undergraduate student teachers, 43 graduate students in education, 80 classroom teachers (26 in primary grades, 45 in grades 4–6, 9 teachers of junior high school mathematics), and 66 other persons. Attempts to classify the answers as showing understanding or complete lack of it were unsuccessful. Likewise, it was not found possible to classify the responses in terms of degree of understanding.

At this stage a multiple choice test of 19 items with four choices per item was made to match identically the data used in the effective questions of the free-answer test. This test was given to 322 students and teachers from Colorado, Illinois and New York City, as well as other places. In this group 76 were teachers of grades 4–6, 67 teachers of junior and senior high school mathematics, 53 teachers of primary grades and 126 "other teachers" not teaching mathematics. The multiple choice material was obtained almost entirely from the responses on the free-answer test.

Dr. Orleans next analyzed the responses on the multiple choice test and compared the findings with the corresponding questions on the free-answer test. Repeatedly, he had to ask himself three questions: Is this answer due to lack of understanding? Is this answer due to an inability to express ideas verbally? Is this answer due to lack of clarity in the question? A fourth one occurred to this reviewer: Is this answer due to a lack of understanding of the term, "understanding"? These four

questions are illustrative of some of the great difficulties in the evaluation process. When time and resources are bountiful the use of individual conferences is sometimes rewarding in dealing with the interpretation of test evidence.

Part of the sample question in the multiple choice test follows:

1. Look at the example at the right. Why is the second partial product (1170) moved over one place to the left?

——(1) In multiplication, each time the multiplicand is multiplied by another number the partial product is moved one place to the left.	234 56 1404 1170 13104
——(2) The partial product is obtained by multiplying by the 5, therefore the answer has to begin under the 5.	
——(3) So that it will make the numbers come out in their proper places.	
——(4) The partial product is 1170 tens.	

This was followed by paragraph explaining why the first three responses were incorrect and the fourth correct.

Some of the most important findings of a quantitative sort were these:

1. About two out of three teachers understand partial products in multiplication.
2. About two out of three teachers know that division is a short-cut for subtraction.
3. About two out of five understand partial products in long division.
4. About two out of five know the meaning of remainder in division.
5. About three out of five know that a fraction can mean a quantity as well as a ratio.
6. About three out of ten understand the changing of fractions to lower or higher terms.
7. About one out of five understand the inverting process in division of fractions.
8. About four out of five understand the moving of the decimal point in the division of decimals.
9. About one out of seven understand the meaning of the remainder in measurement type of division involving division of fractions.
10. About one out of three understand the meaning of per cent.
11. About three out of five know what the base of our number system is.
12. About four out of five know the meaning of decimal fraction.

It is also noteworthy that the differences in the amount of understanding possessed by the four groups are not large.

In supplementing the study the investigator asked the teachers "to indicate (1) the extent of their teaching experience; (2) the amount of college mathematics they had had; (3) their opinions of the extent to which pupils enjoy learning arithmetic; (4) their opinions of their own understanding of arithmetic processes and concepts; and (5) what they think is wrong with the teaching of arithmetic." It is interesting to observe that those with less than five years of teaching experience seemed to understand arithmetic processes and concepts more than those who have been teaching from 5–10 years and that the latter appear to understand as much as those who had more than 10 years of teaching experience. It is also curious that those who had no college mathematics understood almost exactly as much as those who had one or two courses in college. However, those who had three or more courses seemed to understand considerably more than the other two groups. Space limitations prevent the reporting of the other facets of this supplementary investigation. The reader should consult the study himself for this information and for the verbatim responses to the free-answer test. The latter should be of immense value to those concerned with preparing teachers of arithmetic for the type of instruction emphasizing meaning and understanding.

These remarks need to be accentuated by a selection from Dr. Orleans' summary:

"The teacher-education institution may have only an indirect influence on the program of number work in the schools. But it can directly influence the prospective teacher's knowledge and understanding of arithmetic and his preparation for his responsibilities as the person who is to get children to learn about numbers. That our teacher-education programs have not done an adequate job in this respect would seem to be a valid conclusion from the data presented in this report."

NOTES ON THE HISTORY OF MATHEMATICS

Edited By VERA SANFORD
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Sébastien LeClerc's Practical Geometry

There were three men named Sébastien LeClerc. Sébastien LeClerc the Elder was born in Metz in 1637 and died in Paris in 1714. He is said to have been engraver to Louis XIV. LeClerc's son and pupil Sébastien LeClerc the Younger (1676–1763) was a native of Paris. He was prominent as a painter and as a designer of tapestries. Sébastien Jacques LeClerc (1734–1789) called "LeClerc des Gobelins" was presumably of the same family. These three men were associated with the Hôtel des Gobelins, an establishment which had been taken over by Louis XIV to make tapestries and furniture mainly for the court. The life of an artist connected with this institution was a reasonably secure one for the artist received a stipend and living quarters, but the restrictions imposed by those who directed the work must have made such an existence frustrating.

Sébastien LeClerc the Elder was the author of two books which dealt with geometry: *Pratique de Geometrie sur le papier et sur le terrain ov par vne methode nouvelle & singuliere l'on peut avec facilitié & en peu de temps se perfectionner en cette science*,[1] and *Traité de Géométrie*. The second of these, published in Paris in 1664 and 1690, includes material on the plane table and other mathematical instruments in real situations. The reader is referred to Edmond R. Kiely's *Surveying Instruments*, the 19th Yearbook of the National Council of Teachers of Mathematics, in which several facsimile pages of this work appear.

[1] Practice of Geometry on paper and in the field, where by a new and unique method one may perfect himself in this science with ease and in a short time.

PRACTICAL GEOMETRY:

Or, A NEW and EASY

METHOD

OF

Treating that ART.

WHEREBY

The PRACTICE of it is render'd plain and familiar, and the Student is directed in the moſt eaſy manner thro' the ſeveral Parts and Progreſſions of it.

Tranſlated from the FRENCH of

Monſieur *S. LE CLERC.*

THE THIRD EDITION.

Illuſtrated with eighty COPPER-PLATES.

Wherein, beſides the ſeveral Geometrical Figures, are contain'd many Examples of LANDSKIPS, Pieces of ARCHITECTURE, PERSPECTIVE, Draughts of FIGURES, RUINS, &c.

LONDON:

Printed for T. BOWLES, Print and Map-ſeller, in St. *Paul*'s Church-yard; and J. BOWLES, Print and Map-ſeller, in *Mercers-hall, Cheapſide.*

M DCC XXVII.

FIG. 1

The *Pratique de la Geometrie*[2] was printed in Paris in 1668. It was a work of importance as is indicated by the fact that it had at least eight editions in French

[2] The editions are Paris 1668 and 1682, Lausanne 1684, Amsterdam 1692, Berne 1699, also 1700, Paris 1744 and 1774. The third edition in English is London 1727, the fourth 1744. The editions used in the preparation of this paper were the French one of 1682 and the English one of 1727, the illustrations that appear on these pages are from the latter edition. The English edition lacks the dedicatory letter. With a few exceptions, the engravings in this edition are

PROPOSITION V.

Through a given point to draw a line parallel to a given right line.

Let A be the given point thro' which a line is to be drawn parallel to the line B C.

OPERATION.

DRaw at pleaſure the oblique line	A D
upon the point	A
Deſcribe the arc	D E
upon the point	D
Deſcribe the arc	A F
make the arc	D G
equal to the arc	A F
Draw the line required	M N
thro' the points	A & G

Otherwiſe.

Upon the center A deſcribe the arc	E F G
touching the line	B C
without altering the legs of the compaſſes.	
Upon the point H deſcribe the arc	L R I
The point H is taken at pleaſure in the line	B C
Draw the demanded line	O P
thro' the point	A
and touching the arc	L R I

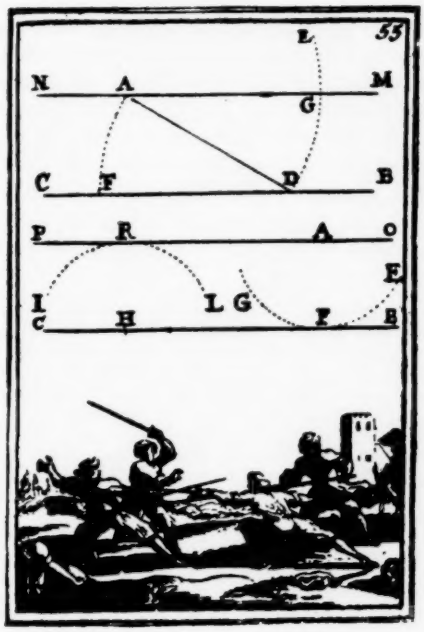

FIG. 2

one being in 1774, and at least four editions of a translation into English. It was plagiarized by Ozanam the author of a book on recreations, with slight alterations in a work of 1699 published both in French and in German. The book was designed with great skill. It is an attractive little volume with a delicate border across the top of each page and with a full page engraving facing each page of the text proper. The third English edition (1727) advertises "Illustrated with eighty Copper-Plates. Wherein, besides the several Geometrical Figures, are contain'd many Examples of Landskips, Pieces of Architecture, Perspective, Draughts of Figures, Ruins, &c." (Fig. 1) You have the feeling that the buildings and "Landskips" are taken from the artist's sketch book. Only two seem to really illustrate the diagram they accompany. Two men are fighting a duel and two more are trying to separate them. The pattern of the weapons suggests two parallel lines cut by a transversal. The text tells how to draw a line through a given point parallel to a given line (Fig. 2). In another case, an argument might be advanced that the construction that is shown could be used in locating the main axis of a garden that leads from the front of a building with a symmetrical façade. The problem is to drop a perpendicular on a line from a point off the line. These arguments fail in the majority of the pictures and the the reader concludes that the object was to embellish the page, not to illustrate the text.

The book is dedicated to the Marquis de Seignelay, a fact which may explain the care with which the book was put together for perhaps this gentleman paid the

close copies of those in the French edition, but in each case the picture is reversed so that the people in them all appear to be left-handed.

costs. The Marquis de Seignelay was the son of Jean Baptiste Colbert (1619–1683), the financier who had become Controller General of France in 1665. His son served as his apprentice and on Colbert's death succeeded to his father's offices. LeClerc had chosen an influential patron.

In the dedication, LeClerc refers to de Seignelay's aptitude for mathematics and says:

> Indeed among the worthy Exercises with which you have been occupied for sometime, does not Geometry share a good part of your hours? And should it not prepare you for the great tasks that await you? Whether it be in peace or in war, civil and military Architecture cannot be planned without it; and the Fortification of Places and the construction of all the public Monuments that preserve the memory of great Princes to posterity can exist only by the help of its rules.

He congratulates his patron on his present achievements and looks forward to even greater deeds for the public good and for the glory of his invincible Monarch.

The text begins with an introduction on Geometry in General. The English text reads:

> Geometry is a Greek word, and in its native signification stands for no more than the measuring of land, but now we mean by it the principal part of Mathematicks, which is a science that has continued quantity for its object. That quantity is call'd continued quantity, which has all its parts conjoin'd; of this kind are all sorts of extension, magnitudes, and dimensions. . . . Geometry is divided into speculative and practical. The former is a science that teaches the mind how to form ideas of, and demonstrate the truth of geometric propositions. The latter or practical Geometry, conducts the hand in writing.

A historical note traces geometry from the practical work in Egypt with this addition—

> But in the process of time, the *Egyptians* applied themselves to more subtle inquiries, and by degrees insensibly there arose from a practice altogether mechanical, a science that now holds the first place among all the others, according to its merit.

The Usefulness of Geometry is treated in two pages. It is necessary to astronomers, to geographers, to architects and to engineers. Then this note.

> Persons of Quality, whose birth engages them to take the field (i.e. go into the army) are oblig'd to apply themselves to this science. It not only serves as an introduction to the art of Fortification, which teaches how to raise proper bastions for the defence of places, and to raise and manage machines that may serve to overturn or make breaches in those of the enemy; but also brings them to great skill and dexterity in the art of war, in forming an army in order for battle, in encamping, dividing the ground for quartering the army, taking maps of counties, plans of towns, forts, and castles, measuring all sorts of dimensions, both accessible and inaccessible, and in forming designs; finally to recommend them as much for their skill and address as for their strength and courage.

A closer translation of the French is "in short to make themselves as competent by their knowledge and skill as by their physical strength and their courage."

LeClerc then states that geometry is built on three sorts of principles: definitions which explain the terms used, axioms which are so evident that it is impossible to question their truth, and petitions or demands so intelligible as to require no explanation.

The pages of definitions equip the reader with a vocabulary without regard to whether the terms will be used later in the book. The author tells about every sort of line and angle he knows. There is a definition of a line, of lines finite and infinite, of an apparent line which is the trace of a crayon or a pen, and hidden or occult lines indicated by two pinpricks. A plumb line goes from high to low without bending to the right or the left and if prolonged indefinitely will pass through the center of the earth. A horizontal line is a line in equilibrium. An oblique line is neither horizontal nor vertical, but is on the bias. Two lines meeting at a point meet either directly or indirectly. If directly, they make one line, if indirectly, they form an angle. Angles are rectilinear, curvilinear or mixed. An oval is a "curvilineal figure describ'd upon several centers, and divided into two equal parts by all its diameters" and "an Ellipse is also a curvilineal figure described upon several centers in the shape of an egg, and has but one diameter that divides it into two equal parts." Compound figures include segments and

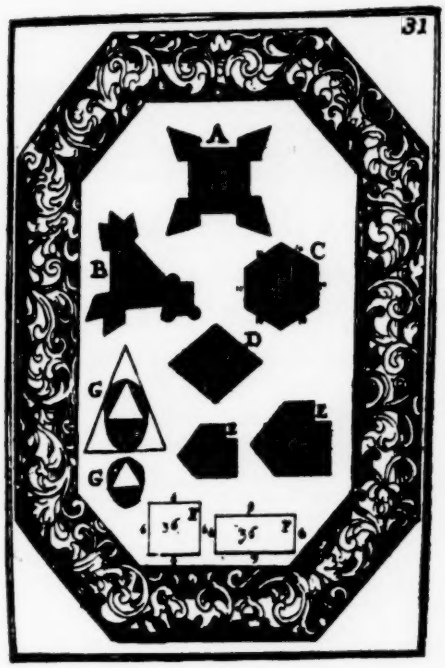
Regular and Irregular Figures.
A A Regular figure is that which has its oppoſite parts ſimilar and equal.
B an Irregular figure is ſuch an one as is compos'd of angles and ſides that are diſſimilar.
E E Similar figures are ſuch as have all their ſides proportional, though one may be greater, equal or leſs than another.
F F equal figures are ſuch whoſe contents are equal, though they may be ſimilar or diſſimilar.
C an Equiangular figure has all its angles equal.
E E one figure is ſaid to be ſimilar or equiangular to another, when all the reſpective angles of the one, are equal to all the reſpective angles of the other.
C D an Equilateral figure is one that has all its ſides equal.
G G Similar curvilineal figures are ſuch as will admit ſimilar Polygons to be inſcribed in them, or circumſcrib'd about them.
31
A
B
C
D
G
G
E
E
F
F

FIG. 3

sectors of a circle, concentric squares and circles, and excentric squares having an angle in common and excenteric circles which are tangent at a point. The shaded areas in the squares and circles suggest the boxwood hedges in formal gardens. Regular figures are those with their opposite parts alike and equal. In other words they are symmetric with respect to a point, and the reentrant polygon shown as an illustration suggests the fortifications that LeClerc's contemporary Vauban designed for Ticonderoga (Fig. 3).

Two pages of axioms follow.

The reader is then given four problems called petitions which show the use of a ruler in drawing a straight line between two points, drawing a circle with given center and radius, and so on.

Having learned how to use a ruler and pair of compasses, the reader progresses to the five books of propositions. These tell how figures are constructed but no explanations are given. They include the usual constructions of a plane geometry text with additions which are noted below. The content is as follows:

First Book of the Descriptions of Lines (perpendiculars, parallels, bisectors with the following innovations)

Upon an angle to erect a straight line which shall incline neither to the right nor to the left. The construction is to bisect the reflex angle formed by the sides of an acute angle.

A circle and a right line touching it being given to find the point of contact.

To draw a spiral line about a given right line.

Between two points to find two others on the same line. The method is to draw two arcs having the same radius using the given points as centers. Then using a smaller radius and the two points of intersection of the first arcs as centers, two other arcs are drawn and their points of intersection are the points required. This note follows "By this method, one can draw in three parts a straight line from point A to point B which could not be done in one part with a ruler which is shorter than the distance between the two points."

Second Book of the Construction of Plane Figures. Constructing various figures having given sides and the like, with these additions,

PROPOSITION XII.

To deſcribe an oval upon a given length.

OPERATION.

Let A B be the length upon which the oval is to be made.

Page 64. DIvide the length given A B
into three equal parts. A C D B
upon the points C & D
with the radius C A
Deſcribe the circles A E F, B E F
upon the interſections E & F
and with the diameter E H
As a radius, deſcribe the arcs I H, O P
A I H B P O will be the oval requir'd.

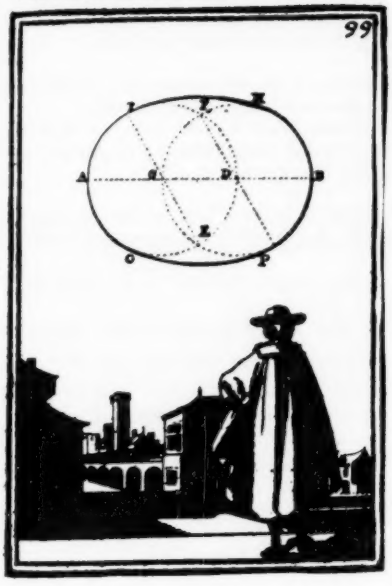

FIG. 4

PROPOSITION XIII.

To deſcribe an oval upon two given diameters.

Let A B, C D be the diameters upon which the oval is to be conſtructed.

OPERATION.

MAke the ruler M O
equal to the greater ſemidiameter A E
upon which mark the length M N
equal to the leſſer ſemidiameter C E
This ruler being thus diſpos'd.
Place it after ſuch a manner upon the diameters A B, C D
that the point N
ſliding along the line A B
the extremity O
may always be in the line C D
carrying along thus the rule M O
Deſcribe the oval with the extremity M

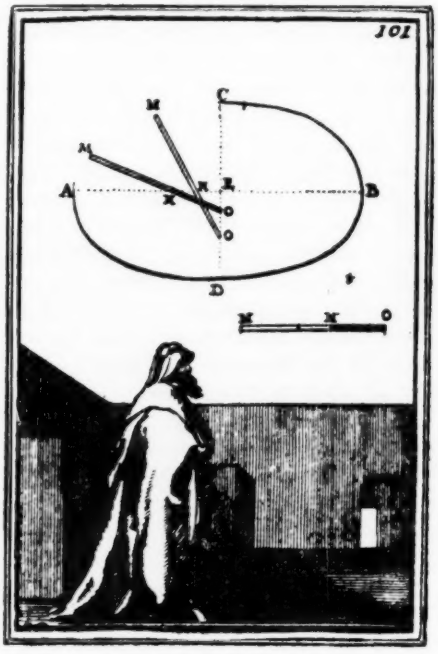

FIG. 5

PROPOSITION VI.

To inſcribe an hendecagon in a given circle.

Let A E F be the given circle in which the hendecagon is to be inſcrib'd.

OPERATION.

Page 58. DRaw the radius A B
Biſect the radius A B
in the point C
upon the points A & C
with the diſtance A C
Deſcribe the arcs C D I, A D.
upon the point I
with the diſtance I D
Deſcribe the arc D O
the diſtance C O
will be the ſide of the hendecagon exact enough for practice.

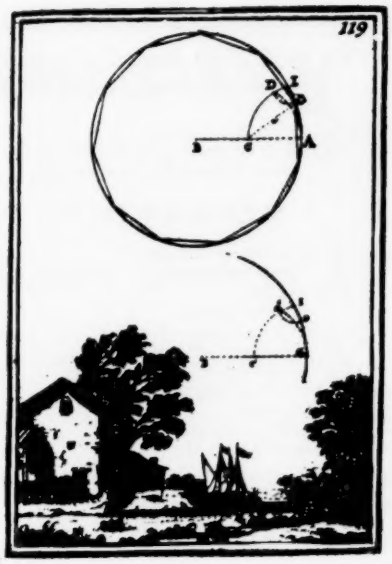

FIG. 6

PROPOSITION XIII.

To inſcribe an equilateral triangle in a ſquare.

Let A B C D be the ſquare in which the equilateral triangle is to be inſcribed.

OPERATION.

DRaw the diagonals A C, B D
upon the center E
and with the diſtance E A
Deſcribe the circle A B C D
upon the point C
with the diſtance C E
Deſcribe the arc G E F
Draw the right lines A F, A G
Draw the right line H I
The equilateral triangle requir'd is A H I.

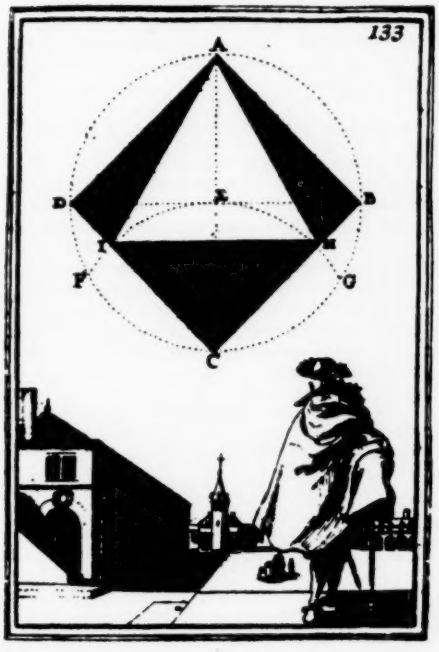

FIG. 7

PROPOSITION IX.

The excefs of the diagonal of a fquare, above the fide being given, to find its fide.

Let A B be the excefs of the diagonal of a fquare above its fide, to find its magnitude.

OPERATION.

Page 50. ERect the perpendicular B C
equal to the excefs A B
Draw the line A C
produced towards D
upon the point C
and with the diftance C B
Deferibe the arc B D
A D will be the fide of the fquare A
the excefs A B
of whofe diagonal A E
above the faid fide A D

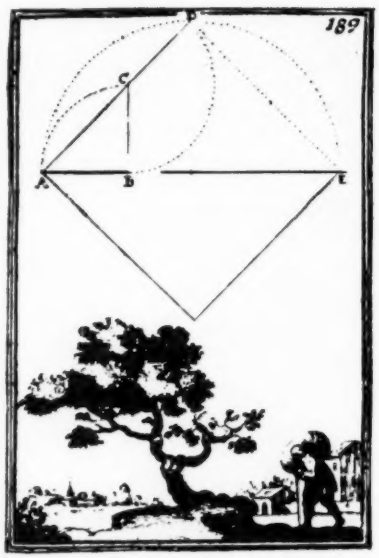

FIG. 8

Upon any line to describe any polygon from a hexagon to a dodecagon. And to describe a polygon of twelve to twenty four sides. These polygons seem to be supposed to be regular figures. To describe an oval upon a given length. This is done by drawing the arcs of circles (Fig. 4). To describe an oval upon two given diameters.[3] (Fig. 5)

Third Book of the Inscribing of Figures

To inscribe polygons in a given circle. This includes polygons of seven, nine, and eleven sides with a note stating that "The distance will be the side of the hendecagon exact enough for practice." As a matter of fact, most of LeClerc's regular polygons were only "exact enough for practice." (Fig. 6) To inscribe in a circle a triangle similar to a given triangle. To inscribe a square in a given triangle, a pentagon in a square, an equilateral triangle in a square (Fig. 7) and in a pentagon.

[3] Nicolas Bion (1709), gives each of these methods of drawing an ellipse (or oval) using the arcs of circles, advising the use of a cord fastened at the two foci if a large ellipse is to be constructed in a garden, and giving both a drawing and description of a trammel to be used in the construction given by LeClerc in the second of these theorems regarding the oval. A modern instrument based on the same principle as LeClerc's construction and Bion's appears in an article by Philip S. Jones, "Mathematical Apparatus," *Multi-Sensory Aids in the Teaching of Mathematics.* (18th Yearbook, National Council of Teachers of Mathematics, p. 214.

Fourth Book of the Circumscription of Figures

To circumscribe a triangle similar to a given triangle about a given circle.

To circumscribe a square about a given equilateral triangle.

Fifth Book of Proportional Lines

Given the sum of the extremes and the mean proportional to distinguish the means i.e. to find the length of the means.

The excess of the diagonal of a square above the side being given, to find the side. (Fig. 8)

To make on a given right line, two rectangles that shall be in a given ratio to each other.

For the "Persons of Quality" whose position makes a knowledge of geometry necessary, this was a convenient way to master a part of the art. A half century after LeClerc's work first appeared, however, a serious student of the applications of geometry would have turned to the encyclopedic work *Traité de la construction et des Principaux Usages des instrumens de Mathématique* by Nicolas Bion, Ingenieur (i.e., designer and constructor) of Mathematical Instruments for the King. This volume is more comprehensive, more thorough, and more scientific, but by no means as appealing as is LeClerc's.

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Edited by WILLIAM L. SCHAAF

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Editor's Note

This is the last of twenty-four issues which we have published during our term as editor of THE MATHEMATICS TEACHER. During this three-year period, the Board of Directors of the National Council has not hesitated to approve those recommendations for our journal which would provide the services needed by our growing organization and which would meet the many and varied needs of the individual mathematics teacher.

On behalf of the Committee on Official Journal, the associate editors, and the departmental editors, we want to express our thanks to the many persons who have given so generously of their time and effort—some by refereeing papers, some by reviewing books, and others by their helpful suggestions and constructive criticisms.

We also want to acknowledge the splendid cooperation of each of the associate editors and the departmental editors. The promptness of delivery of the monthly issues to our members during the last two years was due in no small part to the way each of these editors assumed the responsibilities placed on him. The patience and help of our faithful assistant in our Evanston office, Mrs. Fred M. Wright, must also be given full recognition.

To our successor, Dr. Henry Van Engen, we extend our congratulations and best wishes for the term which begins with the publication of the October issue.

E. H. C. H.

NEWS NOTES

New York University School of Education announces the following courses for its summer session for June 29 to August 7: teaching of arithmetic, applications of mathematics, teaching aids in mathematics education, teaching of junior high school mathematics, teaching of senior high school mathematics, current trends in mathematics curricula and teaching, research investigations in mathematics education.

Under sponsorship of the Fund for the Advancement of Education, the Portland (Oregon) Public Schools and Reed College are cooperating to enrich the **school program for gifted children** by holding **a special seminar in mathematics and physical science at Reed College's** summer session **June 22–August 21.** The seminar is primarily for teachers in the program, but others who are interested will be welcome. Directors of the seminar are Eric T. Bell, Professor-Emeritus of Mathematics, California Institute of Technology; Kenneth E. Davis, Associate Professor of Physics, Reed College; and Robert A. Rosenbaum, Professor of Mathematics, Reed College. Further information can be obtained from the Director of the Summer Session, Reed College, Portland 2, Oregon.

The program at the **Conference on Teaching Arithmetic** to be held at the **University of Wisconsin, July 6 and 7,** will include papers on research and recent investigations in the teaching of arithmetic by national leaders in the field. Provision will also be made for careful discussion of the papers presented. This conference will be of special interest to teachers of arithmetic, elementary school supervisors, principals, secondary school teachers, and college teachers in the areas of mathematics education and elementary education. The School of Education and the Department of Mathematics of The University of Wisconsin and the Wisconsin Mathematics Council are co-sponsors of the Conference. Requests for additional information should be addressed to either Professor John R. Mayor or to Professor Sara A. Rhue, Department of Education, The University of Wisconsin, Madison 6, Wisconsin.

The Third Annual Conference for Teachers of Mathematics at the **University of California at Los Angeles** is being sponsored again by the University in cooperation with the California Mathematics Council for the period **July 6–17.** General sessions include a wide variety of lectures, visitations, and panel discussions while the choice of study groups will satisfy a wide range of individual interests. A special laboratory group provides an opportunity for making teaching devices in addition to demonstrations of how to use models in teaching mathematics Two to three units of college credit may be earned. For further information write to the Department of Conferences and Special Activities, University Extension, University of California, Los Angeles 24, California.

The Third Workshop on Multi-Sensory Aids in the Teaching of Mathematics will be held at **Northwestern University August 3–22.** Four and one-half quarter hours of credit may be earned. For information write to E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois.

Collecting a million pennies for a hospital drive campaign was a project carried on by the 17 pupils of Otto T. Burich's geometry class at the high school in Woodruff, Wisconsin according to a story in the Minneapolis *Tribune* which was called to our attention by R. L. Lokensgard of Winona, Minnesota. The same story was reported in the April 14 issue of *Look* magazine. If the pennies are placed in sand-pile fashion, what are the dimensions of the pile?

Another class project involving "A Million" is reported by Hazel L. Mason of the Grand Prairie (Texas) Junior High School in *Clearing House* for April 1953.

Why Study Math? is a new eight page pamphlet recently published by the Public Relations Department of General Electric Company, 1 River Road, Schenectady 5, New York. Every teacher of mathematics will want a supply for her classes.

A Perpetual Calendar Designed by a General Electric Engineer appeared as the January 1, 1953 poster of the Photo News Service series of the General Electric Company. Free copies of the calendar will be sent to teachers of mathematics by the designer, A. A. Merrill, 1567 Kingston Avenue, Schenectady 9, New York.

Mathematical models in plastic are announced in a recent catalog issued by Gunter Herrmann, Blumenstrasse 14, Hofgeismar, Germany.

A new series of **arithmetic sense-aids** for use in the primary grades are announced by Wilkane Ltd., 42 Commercial Road, Eastbourne, Sussex, England.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

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A SCHOOL BANK

Our eighth grade mathematics class, after the study of a unit in "Arithmetic in Business," organized what they called a school bank or "Corner Junior Bank." The bank was organized about February 25, 1952. At the end of the school term it had proved so successful that the depositors received 93% interest on the money they had invested.

In setting up the bank the eighth graders deposited one dollar each, if they so desired. Deposit slips were filled out in duplicate, one for a receipt for the depositor and one for the bank's records. The money was to be loaned to any student in school. The maximum amount to be borrowed was limited to one dollar; however, the interest on any amount was one cent per day.

The officers for the bank, as elected by the class, were: a president, a vice-president, a secretary, a cashier, and a board of directors. The president and vice-president directed the activities of the bank and gave students permission to borrow. The president or vice-president conferred with the student regarding the amount he wished to borrow and the length of time he wanted the use of the loan. The student carried a card of permission to the cashier, who in turn had him sign this card when she paid him the money. This card stated the amount he was borrowing and the date. When he paid this dept the cashier marked his card paid and dated it. The borrower signed again so there would be no dispute over when or whether the debt had been paid. The secretary kept the records of the bank's business, posting loans and payment from the individual cards to a journal each day. It was easy to check on the bank's condition at any time by checking the secretary's journal. The board of directors helped the other officers any time help was needed, and they collected all delinquent loans.

The bank proved so successful in its three months of operation last year that the eighth-grade class this year wanted to study banking at the beginning of the term so they might organize a bank and begin at the first of the term.

The bank rendered a service to the members of the student body since it provided a means of acquiring a small amount of money when needed. The secretary and cashier received very good practice in keeping records and calculating interest, and realized the greatness of responsibility in handling money. The depositors from the class benefited from the bank since it provided them with the opportunity of investing their money and in realizing a profit on their investment. During the year the depositors figured what per cent interest they were making, and at the end of the school year they calculated their interest for the year and the cashier paid each depositor the amount deposited plus the interest he had realized on it. Each year, then, the bank can be organized anew.

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An Experiment with Students Failing in General Mathematics

At the end of the first semester eleven students were taken out of the tenth grade general mathematics classes and put in a class by themselves. Their I.Q.'s ranged from 73 to 96.

The first problem was to change their attitude toward mathematics. This was accomplished by the following devices.

1. Much of the class time the first week was spent in conversation. The students were encouraged to talk about themselves and their hobbies. This gave clues to the thought problems that might interest these students.
2. The work was started with a unit on estimation. The students discovered that there were many places where estimating was important. (For example, parking a car, passing a car, etc.) They practiced estimating and found it fun. They learned to estimate distance by pacing.
3. All criticism was constructive. It was always the number right—never the number wrong.
4. No outside assignments were given for two weeks. At the end of two weeks the students suggested that they be given assignments so that they could make more progress. The assignments were always short and great care was taken that each student knew how to work the problems before he left class.

Remedial work was accomplished by the following methods.

1. The Brueckner Diagnostic Tests in Whole Numbers, Fractions, and Decimals were given. The students were told that they would have a chance to try the tests again later in the semester.
2. The work was based upon the errors made in these tests. The text was used very little. Meaning, rather than mere mechanical skill, was emphasized.
3. Word problems were simple but practical.

The results of the semester's work can be summarized briefly.

1. The students' attitude toward mathematics was changed. As one student said, "This stuff isn't bad if you know what you are supposed to do." When one boy was arrested for speeding past a school, the problem of how long it actually takes to stop a car was studied. The students became so interested in these problems that they came in after school to make sure that they were working them correctly.
2. The students gained confidence in their ability to manipulate numbers and to work simple word problems.
3. The number of problems missed by the entire class on the Diagnostic Tests in January, compared to the number missed in April, is given below:

	Whole Numbers	Fractions	Decimals
January	353	575	342
April	216	269	197

Some individuals made marked gains. Some reduced their errors by more than 50%. One boy with an I.Q. of 73 reduced his errors in whole numbers from 37 to 16, in fractions from 105 to 12, and in decimals from 34 to 18. While they still made errors, they were able to find their mistakes without help.

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Adjusting First-Year College Mathematics for Special Interest Groups

Montana State College is one of the six units of the Greater University of Montana. It is a land grant college with a pres-

ent enrollment of about 2000 four-year students. It has divisions of agriculture, engineering, household and applied arts, science, and the graduate division. Other branches such as law and forestry are located at one of the other units. The enrollment is a little heavier in the division of science than in the other three undergraduate divisions. Engineering stands second in this respect, with the other two divisions not far behind. With such an emphasis on mathematics and science, it is a natural step to the preparation of teachers for teaching in these fields in the secondary schools of Montana. However, the preparation of teachers is not limited to these fields, but includes majors or minors in all of the subjects usually taught in high schools. The class of 1956 has an enrollment of 667. Only three curricula do not require some mathematics, and from time to time two of these do allow or encourage students to elect mathematics. The department of mathematics at Montana State College is not a degree-granting department on the undergraduate level. This means it is a service department for those departments who feel the need of its services.

The preparation of freshmen in mathematics varies from no high school mathematics to college algebra and trigonometry. Most of the freshmen have had high school Algebra I and plane geometry—some may have had only general mathematics. Those interested in engineering usually have had advanced high school algebra and trigonometry. Many are entering college with an interest in engineering who have not had solid geometry.

The offerings in mathematics for freshmen are quite varied. High school algebra and plane geometry are available for those who have not had these courses in high school, or who took them so long ago that a refresher is needed. Advanced high school algebra and solid geometry are offered for those who wish to take mathematics that require these courses as prerequisites or, in the case of advanced algebra, for those who have taken it but feel the need of review. The first three mentioned are 5-hour courses without college credit; the solid geometry is a 3-hour course without college credit. This fall there were about 25 in Algebra I, 60 in Algebra II, 25 in plane geometry, and 80 in solid geometry. A three-quarter course in shop mathematics (three hours per quarter, but no college credit) is offered for those taking short courses in agriculture and allied subjects.

A course, intermediate mathematics, is available and is designed for home economics students and students in nursing. It is given in the fall quarter before their first quarter of chemistry. Intermediate mathematics is designed to give a review of arithmetic and algebra such as will be needed in first-year science courses. It is planned on the assumption that students have had two years of high school mathematics. It is a four-hour course and is the only college mathematics taken by these two groups at the present. Only those students with a placement score below five are enrolled in the course. Forty-three were enrolled in the fall of 1952.

A two-quarter course, elementary analysis, is designed for students in agriculture, applied science, botany and bacteriology, zoology and entomology, and in one option in commercial science. Elementary analysis covers the topics usually given in college algebra and trigonometry, but not as completely. The third quarter in the sequence is elementary statistics. This sequence of courses requires four hours per quarter. The elementary statistics is taught by three hours of lecture and one (credit-) hour of laboratory. No attempt is made to segregate the different groups into sections. In a sense this sequence is a terminal sequence. There are a total of 175 students enrolled in elementary analysis this fall, about 125 of them being freshmen. The agriculture curricula now call for one quarter of this course before the student starts chemistry.

Introductory college mathematics is

designed primarily for those interested in engineering, mathematics, physics, and chemistry. It is also used in certain military science options. It is a three-quarter, 5-hour per quarter, course, covering college algebra, trigonometry, and analytical geometry. Some calculus is introduced early and used throughout the year. It is the course that freshmen take who desire to pursue their mathematical training beyond the first year. No attempt is made to arrange sections for special interest groups, nor is it thought desirable. There are about one hundred eighty enrolled in this course for the fall quarter.

During the first class hour of the courses in advanced high school algebra, elementary analysis, and introductory college mathematics, the students are given a short test—adapted to the particular class—which is purposely made so easy that anyone who fails it is obviously in the wrong course. The next day these people are advised to see their class advisors about shifting into a more desirable mathematics course. Then each instructor confers, usually by telephone, with the advisor. Sometimes the student does not feel the need for changing. Usually in about two weeks they will initiate the move—and feel happier about it. It has been necessary for the mathematics department to make the advisors aware of what the department is trying to do. Most of the courses have prerequisites, but the mathematics department has found it expedient to hold conferences with class advisors in certain cases. In several cases the prerequisites have been waived, and in all cases a better understanding of what the department is trying to do has resulted. This understanding has taken time, but it is paying dividends in satisfaction to the student and ease of instruction to the department.

The method of teaching the various courses is not the same. The department attempts to assign teachers to teach the so-called "high school" subjects who have had experience in teaching those subjects in high school. Some use is made of persons who are planning on entering high school teaching when they have finished their training. In the latter case some experienced teacher is appointed to supervise this work. An attempt is thus made to help bridge the gap between high school and college mathematics.

In the teaching of all courses much use is made of blackboard work, tests, home work, help sections, make-up tests, etc. The lecture method is used extensively in some courses. In the so-called "high-school" mathematics courses, intermediate mathematics, and elementary analysis, an attempt is made to hold lecturing to a minimum. Board-work receives much play. Frequent tests in all courses are the accepted policy. Help sections are a common feature, particularly in four-credit courses. These can usually be scheduled on the fifth day at the same hour as the regular class. Attendance is voluntary. Points of difficulty are taken up. Additional drill is given. Homework is stressed. It is picked up and graded either by the instructor or by a reader. Several instructors feel it is quite desirable to read the papers themselves. It gives a closer contact with the troubles of the class. Frequent make-up tests are given. These are given on the students' time and prove to be an excellent teaching device. In the elementary analysis, less stress is put on formal proofs and the more abstract phases of mathematics. Many topics are omitted that are included in the introductory college mathematics.

In teaching elementary analysis and intermediate mathematics, it has been found to be very helpful to follow an explanation of some new work by having the class do one or two problems in class or on the board. Their troubles soon become apparent and help can be given them at once.

In the introductory college mathematics the use of a book with a unified approach has been felt desirable. This seems to prevent some who have had a very good high school preparation from feeling bored at the beginning. Further, it tends to

present mathematics as a unit instead of as separate subjects. And the textbook that has been used at Montana State College heads forward into the second year of mathematics much more easily. It is felt that there may be a little higher mortality in the freshman year by this approach than under the conventional approach. But it is probably better to have the mortality in the first year than in the second year. It seemed to occur in the second quarter of calculus under the former plan.

A type of homogeneous grouping on a limited scale has been tried in elementary analysis the past two years. Each fall there have been two sections of the course scheduled at the same hour. After the first hour test, given at the end of the first three weeks, those making a grade above 70 on the test, from both sections, are put into one section, and the others are put into a second section. Those in the lower section are given more drill and do not cover the more difficult problems. The other section is given problems and a few topics, like the solution of simultaneous linear equations by determinants, that are not covered by the slower section. This practice has led to fewer failures. If this grouping could be done earlier, the results might be more desirable.

In all of the efforts to care for the different interest groups, the department feels that the topnotch student is the one that does not find much done for his welfare. The method of selection does aid him some, but it seems to be almost impossible without some special provisions to get at the excellent student. The department has attempted to encourage students with excellent high school preparation and an excellent placement score to challenge some course—say trigonometry—and thereby gain a quarter with his mathematics. On occasion special instruction has been offered such students. Last fall an attempt was made to get a number interested in such a program. For a number of reasons the project has not been as successful as one might hope.

In summary, there are certain points that seem outstanding:

1. In any program of adjustment of mathematics to special interest groups as outlined in this paper, there is need of a testing and guidance program to be utilized at registration time.
2. Since the guidance of choice of courses is done by the class advisors, it is important that they be informed about the program of the mathematics department. This calls for cooperation of the department and the class advisors.
3. Different interest groups require different methods of teaching mathematics.
4. Special courses seem desirable for those interest groups of students who are taking only one or two courses in mathematics.
5. High school mathematics courses seem to be in considerable demand for college freshmen.
6. In such a program as is outlined in this paper, trailer courses are inevitable.
7. An objective of any and all mathematics courses should be understanding of the steps and processes involved. Manipulations and shortcuts should follow understanding.
8. This paper does not mean to imply that the problem is solved at Montana State College. Neither is it meant that the steps discussed herein are unique or are the best solutions. The paper does call the attention of those interested to the fact that the department is aware of the problem and is trying to find a solution.
9. The department takes the incoming freshman where he is and attempts to get him where he wants to go.

ADRIEN L. HESS
Montana State College
Bozeman, Montana

APPLICATIONS

Edited by SHELDON S. MYERS

Department of Education, Ohio State University, Columbus, Ohio

Ar. 28 Gr. 6–9 *Steam Carriage to Automobile* (By CHARLES C. WEIDEMANN)

In 1865 Parliament required that "all mechanically propelled vehicles" be restricted to a maximum of 4 miles an hour in the country and 2 miles an hour in towns. Drivers of power vehicles were required to have a man walking in front with a red flag. It had to be driven "by not less than three persons." In 1880 the steam carriage traveled 66 miles in 3 hours on the level or _____ miles per hour. In 1884 the gas engine became practical. Daimler applied it to a bicycle; later this became the four-wheeled motor car.

In 1885, Benz built a gas engine rated at $\frac{3}{4}$ of a horsepower. If one horsepower equals 33,000 foot-pounds per minute of power, how many foot-pounds per minute did this engine rate? _____

The early motor car (1893, France) was restricted in Paris along the Seine River to a maximum of 12 kilometers per hour or _____ miles per hour. The speed limit in the country was 20 kilometers per hour or _____ miles per hour. In 1895 an auto race was won with an average speed of 15 miles per hour. The motor was rated at 4 horsepower, or _____ foot-pounds per minute or _____ foot-pounds per second. In 1897 an 8 horsepower car won the Paris-Amsterdam race at an average speed of 30 miles per hour. From 1895 to 1897 the per cent increase of speed over the 1895 record was about _____ %.

In 1903 an 80 horsepower motor averaged 53 miles per hour. What is the ratio of the horsepower of this motor to that of 1897? In the same year a 100 h.p. car raced $1\frac{3}{4}$ miles in one minute. How many miles per hour is this? _____ The R.P.M. (revolutions per minute) of the fastest motors then were 800 or _____ R.P.S. (revolutions per second).

In one race of 732 miles 22 inner tubes were used, or an average of one tube for every _____ miles.

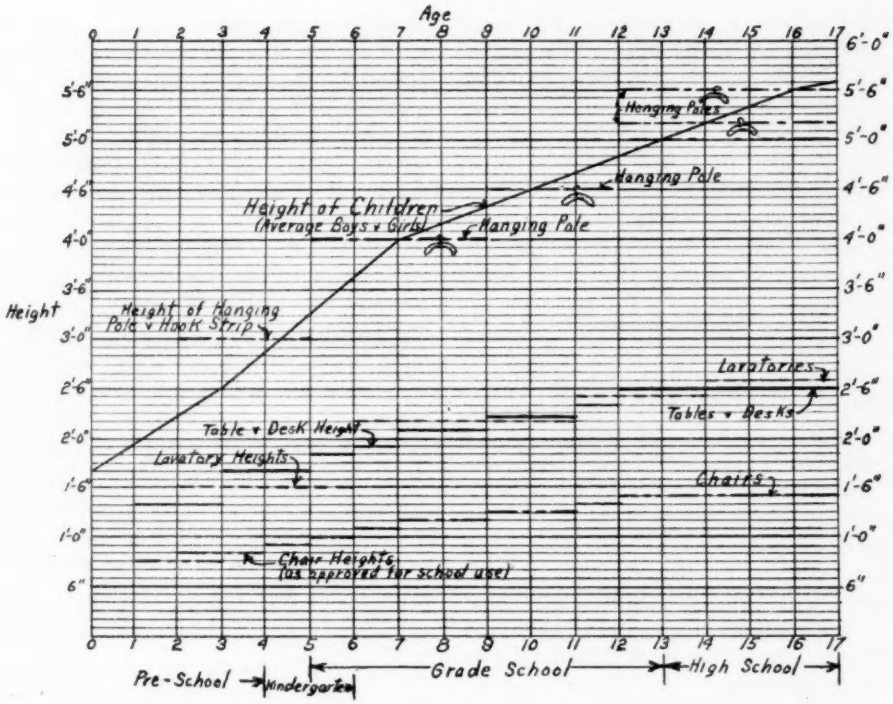
So rapid has been the development of the motor car that in 1937 we are faced with such problems as the following:

1. The record of Campbell over a measured mile was at the rate of 301.13 miles per hour. How many miles per minute? _____ How many miles per second? _____ How many seconds per mile? _____ How many feet per second? _____
2. Jenkins averaged 148.63 miles per hour for a continuous run of 48 hours. How many miles did he cover? _____ If the motor trip across the United States is about 3550 miles, how many trips would the mileage covered by Jenkins make? _____ At his rate he would cross the United States in about _____ hours.
3. A certain stock car travelled 2000 miles at an average speed of 68.54 miles per hour. How many hours and minutes to travel the 2000 miles? _____ How many miles per minute? _____ How many feet per second? _____

Department Editor's note: The Contest Board of the American Automobile Association publishes a mimeographed manual containing rather complete performance figures for stock car races and dirt tracks throughout the country, including the great Memorial Day races.

Ar. 29 Gr. 6–10 *Use of the Growth Curve for Determining the Height of School Furniture*

In the authoritative book *Architectural Graphic Standards* by C. G. Ramsey and H. R. Sleeper, 4th Edition, John Wiley and Sons, 1951, p. 511 there is an interesting use of the average growth curve for boys and girls. The copy of the curve reproduced here shows how the standard heights for certain items of school furniture is adjusted to the heights of growing boys and girls. One question that might be raised is why the heights of various



Graph showing heights of children and their furniture.

(Reprinted with permission from Ramsey and Sleeper, *Architectural Graphic Standards*, 1951, John Wiley and Sons, Inc.)

furniture items are shown by horizontal segments instead of a continuous curve, like the growth curve. A question in the writer's mind, as yet unanswered, is why the growth curve for children does not show a spurt for adolescence after age 10, 11 or 12. Some teachers might have their classes find their average height, determine on this curve how high their chairs and desks should be, and then check by actual measurement. This would provide experience in measurement, approximation, and graphical interpretation in a practical setting.

Ar. 30 Gr. 8–9 *More About Installment Loans*

C. N. Mills, Illinois State Normal University, Normal, Illinois, sent the following application showing how certain topics can be considered at a lower grade level.

More About Installment Loans

In the November 1952 issue, Frank C. Gentry has presented a clear discussion of paying off a loan by equal periodic payments. For several years we have given this special problem in our junior high school classes. The same formulas have been developed by inference using the process of multiplication in algebra. Only a fifty-period four-place compound interest table is used. During the development of the desired formulas, the teacher has an opportunity to point out to the pupils the need of further study in advanced algebra for the proof of the formulas.

Using the following notation, which is more familiar to the pupils, the two main formulas are developed.

Let
P be the original loan,
r be the interest rate per installment period,
p be the periodic payment
n be the number of payments,
P_n be the balance after n payments.

$$P_n = P(1+r)^n - p\left[\frac{(1+r)^n - 1}{r}\right] \quad (1)$$

$$(1+r)^n = \frac{p}{p - Pr} \quad (2)$$

By using only the compound interest table, but without using logarithms, the pupil gets much needed practice in the fundamental processes of arithmetic.

Example 1. A loan of \$3000 is to be paid by annual payments of \$300. Each payment includes the interest and the remainder is to reduce the principal. The annual rate of interest is 6%. Find (a) the balance at the end of 10 years and (b) the number of years it will take to pay off the loan.

Solution. Using formula (1)

$$\text{(a)}\quad 3000(1.06)^{10} - 300\left[\frac{(1.06)^{10} - 1}{.06}\right],$$

$$3000(1.7908) - 300\left[\frac{1.7908 - 1}{.06}\right],$$

$$\$3000(1.7908) - \$300\ (13.18)$$

$$= \$1418.40.$$

(b) Using formula (2)

$$(1.06)^n = \frac{300}{300 - 180} = 2.5000.$$

The number in the 6% column nearer to 2.5000 is 2.5404. Opposite this number in the n column is 16. Hence it will take about 16 years to pay off the loan.

Example 2. A loan of \$4000 is to be paid by monthly payments of \$60, at 6% annual interest, or ½% monthly interest. Find (a) the balance at the end of four years and (b) the number of years it will take to pay off the loan.

Solution. Using formula (1) where $n = 48$.

$$4000(1.005)^{48} - 60\left[\frac{(1.005)^n - 1}{.005}\right],$$

$$4000(1.2705) - 60\left[\frac{1.2705 - 1}{.005}\right],$$

$$\$4000(1.2705) - \$60(54.10) = \$1836.00.$$

(b) Using formula (2)

$$(1.005)^n = \frac{60}{50 - 60} = 1.5000.$$

It is impossible to locate the number 1.5000 in the 50-period table. Extract the square root of each side of the above equation, which gives 1.2247. Find the square root by the division method. In the ½% column 1.2247 is nearer to 1.2268. Opposite this number in the n column we find 41. Hence $n/2 = 41$, and $n = 82$. It will take about 82 months or about 7 years to pay off the loan.

Example 3. Find $(1.06)^{72}$, using only a 50-period table.

Solution. $(1.06)^{36} = 8.1473$. Square each side of this equation. $(1.06)^{72} = 66.3787$.

Example 4. Given $(1.005)^n = 2.5796$. Using a 50-period table find n.

Solution. Extracting square root of each side of the given equation, gives $(1.005)^{n/2} = 1.6061$. Extracting square root again, gives $(1.005)^{n/4} = 1.2673$. In the ½% column this number is nearer 1.2705, which is opposite 48 in the n column. Hence $n/4 = 48$, and $n = 192$.

C. Al. 8 *Loan Amortization*

The application on Installment Loans by Frank Gentry appearing in the November issue of this department not only stimulated the above contribution, but also the one that follows by Adrian Struyk, Clifton High School, Clifton, New Jersey.

Loan Amortization

The interesting situation presented by F. C. Gentry in these columns in the November 1952 issue of Mathematics Teacher is an example of a type of problem which has been put to the writer more

than once, both in and out of the classroom. No doubt others have had the same experience. An alternative way of dealing with the matter, involving no induction, but applying the principles of geometric progression, is offered herewith. It is suitable for use with classes in intermediate algebra.

The problem may be stated as follows. An interest-bearing debt is cleared by making payments at the ends of successive equal time periods. The payments are equal, excepting the last one, which may be less than the others. Part of each installment pays interest, for the period ending, on the unpaid balance; the rest reduces the unpaid balance. Show how to deal with questions likely to arise when arranging for or when paying off such a debt.

Our discussion will be based upon consideration of the debt-reducing portions of the successive payments. The principal feature of these debt reductions is that they are in geometric sequence. After establishing this fact the desired formulas are easily obtained. The following notation is convenient.

B_0 = the principal (dollars),
i = the interest rate (dollars per dollar per period),
I = the amount of each of the equal installments (dollars),
n = the number of installments of amount I to clear the debt,
d_r = the debt-reducing part of the rth installment,
D_r = the total debt reduction effected by the first r installments,
I_r = the total interest paid in the first r installments,
B_r = the unpaid balance after the first r installments.

In order to initiate reduction of the debt the value of I must exceed iB_0, the interest due on B_0 at the end of the first period. Then

$$d_1 = I - iB_0.$$

Since part of each subsequent installment further reduces the remaining debt, the interest to be paid also decreases with successive installments. The amount by which the interest portion decreases is added to the debt-reducing portion to keep I constant. Since the $(r-1)$th payment reduces the debt by d_{r-1} the interest id_{r-1} is not included in the interest-paying part of the rth payment, but is added to the debt-reducing part. Thus

$$d_r = d_{r-1} + id_{r-1}, \text{ or}$$
$$d_r = (1+i)d_{r-1}.$$

This equation shows that the sequence $d_1, d_2, d_3, \cdots d_r, \cdots$ is a geometric progression with common ratio $(1+i)$. Applying well-known formulas we have immediately such results as

$$d_r = d_1(1+i)^{r-1} = (I - iB_0)(1+i)^{r-1},$$

$$d_1 + d_2 + \cdots + d_r = D_r$$
$$= (I - iB_0)\frac{(1+i)^r - 1}{i},$$

$$d_{r+1} + d_{r+2} + \cdots + d_{r+s} = D_{r+s} - D_r$$
$$= (I - iB_0)(1+i)^r \frac{(1+i)^s - 1}{i}.$$

From these it follows that

$$B_r = B_0 - D_r = B_0 - (I - iB_0)\frac{(1+i)^r - 1}{i},$$

so that

$$B_r = \frac{I - (I - iB_0)(1+i)^r}{i}. \quad (1)$$

$$I_{r+s} - I_r = sI - (D_{r+s} - D_r),$$

making

$$I_{r+s} - I_s = sI - (I - iB_0)(1+i)^r \frac{(1+i)^s - 1}{i}. \quad (2)$$

As defined in the notation list the number n makes $B_n = 0$. Taking $r = n$ in equation (1)

$$I = (I - iB_0)(1+i)^n.$$

(*Continued on page 376*)

AIDS TO TEACHING

Edited By

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BOOKLETS

B. 146—You Will Like Geometry

Museum of Science and Industry, Jackson Park, Chicago 37, Illinois.

Booklet by Karl Menger; 6″×9″; 34 pages; $.13 includes postage.

Description.—Profusely illustrated with drawings, this booklet gives concise statements on the nature of the geometric world. However, the geometric world of this booklet is much wider than that found in an Euclidean geometry textbook. The geometry of lines, angles, and planes is extended to topics such as closed curves, square circles, and universal curves. Methods of trisecting an angle, squaring a circle and a variety of related constructions are illustrated. Descriptions of regular polygons and polyhedrons are extended to show the development of two and three dimensional compounds. Advanced topics such as duality, isomorphism, translations and reflections, and groups of operations are discussed and illustrated in concrete terms that will readily be understood by capable high school students.

Appraisal: Although this booklet was written as a guide book for a geometry exhibition at the Museum of Science and Industry, it can be a guide book for superior high school students into a variety of geometric topics. Its concise style, varied drawings, and simple illustrations present a wide scope of topics in a few pages. Despite its brief treatment it attains remarkable clarity and simplicity. It will furnish a great many ideas for projects, topics, or exhibits for interested, capable high school mathematics students.

B. *147—The Pegasus*

Fairchild Engine and Airplane Corp., Hagerstown, Maryland.

B. 148—Boeing Magazine

Boeing Airplane Co., 7755 E. Marginal Way, Seattle, Washington.

Magazines: 8½″×11″; free to libraries.

Description: These magazines are the house organs of these companies. As such they are concerned with the products, policies and people of the company. Contents include articles on new aircraft being built, descriptions of aircraft in service and the development of new products.

Appraisal: Although these publications primarily advertise a product and a company, their profuse illustrations often can be used for bulletin board displays. Descriptions of airplanes and flights give material which illustrates applications of mathematics.

B. 149—Precision, A Measure of Progress

Booklet; 5½″×8½; 64 pages; 1952; Free in limited quantities.

Department of Public Relations, General Motors, Detroit 2, Michigan.

Description: This booklet sketches the history of our units of linear measure and shows how the increase in accuracy has influenced the products of science and industry. It describes precision measuring devices such as the micrometer, Johannsson blocks, and electronic gauges. It also

describes ingenious ways in which air pressure, light waves, and radar are used in precision measurement. Frequent illustrations are given to show the relationship of mass production and refined methods of measurement. A glossary of weights and measures and conversion tables is included.

Appraisal: Attractively illustrated and written in a direct yet interesting style, mathematics teachers will find this booklet one of the most useful free publications now available. Although frequent application is made to the automobile industry there is no advertising in the entire booklet.

B. 150—Athletic Field and Court Diagrams

Lowe and Campbell Athletic Goods; 225 N. Wabash Ave., Chicago 1, Illinois.

Booklet; 32 pages; $4\frac{1}{2}'' \times 6\frac{1}{2}''$; free

Description: This booklet contains diagrams of fields and courts of all common American games. Each diagram gives the official dimensions with suggestions for construction or layout where necessary. Weight charts, official ball specifications, metric and English distance equivalents, and tournament drawings are also discussed.

Appraisal: This publication will furnish the mathematics teacher with material of real interest to youth. It can be used to suggest projects for field work or data for problems, for example, "Lay out a baseball field according to specifications" or "If the 100 yard dash is equivalent to 91.44 meters, how many meters in the 220 yard dash?"

B. 151—Computations with Approximate Numbers

School Science and Mathematics, P. O. Box 408, Oak Park, Illinois.

Booklet; 17 pages; $6'' \times 9''$; $.25.

Description: It is the purpose of this article by W. A. Gager to build an understanding of the use of approximate numbers. It first discusses terms such as units of measurement, significant digits, accuracy, error, precision of measurement, and rounding off numbers. Rules for computing with approximate numbers are given for addition, subtraction, multiplication, division, square root, and regular measurement.

Appraisal: Much confusion exists with respect to proper procedures to use in computing with approximate numbers. This article not only gives specific rules to follow but also shows why the rule is reasonable.

B. 152—Your Mathematics Notebook

Scott, Foresman and Company, 433 E. Erie St., Chicago, Illinois.

Pamphlet; $8\frac{1}{2}'' \times 11''$; 4 pages; free.

Description: This advertising leaflet which is published several times a year contains articles on the teaching of mathematics and is illustrated by photographs and cartoons. The articles are usually reprints from periodical literature or textbook guides on topics such as concrete materials, reading difficulties, learning activities. Frequently photographs or drawings from the publishers' textbooks are used to illustrate the articles.

Appraisal: The materials of this publication will be of interest and help in the teaching of mathematics even though they are selected to advertise the textbooks of the publisher. The articles frequently describe activities, materials, and references that will be helpful in suggesting new ways to teach old topics.

B. 153—The Earth is a Magnet (6 pages)

B. 154—The Secret of Flight (4 pages)

Oklahoma Aviation Commission, Route 6, Box 249, Oklahoma City, Oklahoma.

Pamphlets; $8\frac{1}{2}'' \times 11''$; free.

Description of B. 153: This pamphlet uses cartoons to illustrate the magnetic compass and how it is used to locate positions on the earth's surface. It illustrates and discusses briefly topics such as magnetism,

magnetic variation, elements of navigation, aviation maps, and radar. Mathematics is involved in computing the compass course, map projection, and the waves of radar.

Description of B. 154: This is a cartoon presentation of basic forces and principles of flight. It illustrates and discusses briefly aerodynamics and airplane control. The section on aerodynamics covers air pressure, lift, drag, angle of attack and the wind tunnel. Airplane control shows the function of the elevators, ailerons, rudder, and how these parts are operated by the pilot.

Appraisal of B. 153 and B. 154: These cartoons are clever illustrations of the problems of flight presenting a very great amount of material in few pages. To do this the cartoons are necessarily small and the sequence somewhat confusing.

B. 155—Financial Security Topics for Teachers

Educational Division, Institute of Life Insurance, 488 Madison Avenue, New York 22, New York.

Pamphlet; $8\frac{1}{2}'' \times 11''$; 4 pages; free.

Description: This bulletin is published periodically throughout the school year for the Committee on Family Financial Security Education. The committee sponsors workshops during the summer at several institutions. These workshops are financed by the Institute of Life Insurance and provide many scholarships covering tuition and travel allowance for teachers who wish to prepare themselves for more effective educational service in the field of family financial security in their schools. The bulletin provides information about these workshops, announcement of available resource units which have been prepared at the workshops, new teaching materials in the field of financial security, and articles related to this field.

Appraisal: This publication will be a good source of reference for current materials for the general mathematics or consumer mathematics teacher.

B. 156—Small Loan Laws of the United States

Consumer Education Department, Household Finance Corporation, 919 North Michigan Avenue, Chicago 11, Illinois.

Booklet; $6'' \times 9''$; 32 pages; free.

Description: This booklet describing the origin of small loan legislation, discusses small loan laws that are now in force in many states, and explains fair interest rates. It tells about the principal provisions that make up a good small loan law and shows the great need for effective small loan legislation to protect the consumer.

Appraisal: Although containing no mathematical problems, it contains data and loan rates that can be used as the basis for problems in a unit on consumer credit.

B. 157—Consumer Credit Facts for You

Consumer Education Department, Household Finance Corporation 919 North Michigan Avenue, Chicago 11, Illinois.

Booklet; $6'' \times 9''$; 32 pages; $.10.

Description: This bulletin is a survey of the various types of credit available to the consumer, such as installment loans, single payment loans, charge accounts and service credit. The booklet outlines the characteristics of each type and the source of credit. State and Federal legislation regulating consumer credit and small loans is described. Examples are given to illustrate how to determine the actual interest rate on various types of credit. The reader is urged to use consumer credit wisely by using credit only when necessary, in a reasonable amount, and in terms of the best credit bargain.

Appraisal: This booklet is written by W. P. Mors of the Bureau of Business Research, Western Reserve University, and appears to be an unbiased study of consumer credit. It contains information, data, and problems that are suitable for general mathematics courses in grades 9 through 12.

B. 158—How Much Government

The Conference Board, Division of Education Planning, 247 Park Avenue, New York 17, New York.

Booklet; $7\frac{1}{2}'' \times 10''$; 32 pages; $1.00.

Description: This is a collection of weekly charts issued by the Conference Board on government spending, taxation, and financing. The charts are on topics such as the public debt, our national wealth, amount and source of taxes, government payroll, government expenditures, welfare expenditures, and proposed reductions in government spending. Each chart is accompanied by a page of interpretation of the charts.

Appraisal: The charts of this publication are of varied types and printed in attractive colors. They are published to support the point of view that the role of government in our society has expanded to the danger point. It will furnish excellent illustrations of different types of graphs.

B. 159—O.U. Mathematics Letter

Professor Richard Andree, Department of Mathematics, University of Oklahoma, Norman, Oklahoma.

Newsletter; printed; free to senior high school teachers.

Description: This newsletter, sponsored by the University of Oklahoma chapter of Pi Mu Epsilon, is directed toward the high school mathematics student. Its purpose is to stimulate interest in mathematics in high school as well as in college. It presents unusual topics in mathematics such as the binary system and angle trisection, unusual problems and puzzles, and news notes about courses, lectures, scholarships and research. Students are invited to submit solutions to the problems. The correct solution and the names of the students sending in correct solutions are listed in subsequent issues. Since only one letter is sent to each senior high school teacher requesting it, permission is granted to schools to reproduce any material published in the letter. Professor Andree requests that if several teachers from the same high school desire copies, they submit all requests in the same letter so he may address one envelope for the several copies.

Appraisal: The success of this publication is indicated by the increase in circulation to 2,500 in less than a year. Its success is due to the fact that it is meeting the need of secondary school teachers for material to motivate and supplement senior high school courses.

B. 160—Aviation Education (Curriculum Bulletin Number 6)

State Department of Education, Montpelier, Vermont.

Booklet; $8\frac{1}{2}'' \times 11''$; 57 pages; 1951; $.36.

Description: There are suggestions in this curriculum bulletin for aviation units for grades one through twelve. Each unit includes objectives, suggested approaches, concepts to be developed and activities to be planned by children and teacher. Many of the units suggest the correlation of the unit with various school subjects. A bibliography of aviation stories according to grade level is included. Brief background material on how the airplane flies and the place of aviation education introduces the unit outlines.

Appraisal: This bulletin will be particularly helpful for the elementary school teacher. The problems and activities suggested can provide a basis for planning units that will integrate topics from several areas. It will not furnish the teacher with the necessary background material.

B. 161—Professional Opportunities in Mathematics

Mathematical Association of America, University of Buffalo, Buffalo, New York.

Booklet; 24 pages, $7'' \times 10''$; $.25 (single copies), $.10 (ten or more).

Description: This report is designed to aid the young student who is considering mathematics as a career. It describes the

types of work a mathematician or person with considerable mathematical training does in universities, in business, and in government; and suggests the types of college preparation needed for these various positions. The report covers the fields of mathematics teaching (high school and college), applied or mathematical statistician, industrial laboratory mathematician, government mathematician and the actuary. In each field the character of the work is described, suggestions are made concerning college courses which are suitable, and a rough estimate is given of the salaries now paid (January 1951) to those who are successfully employed in that kind of work.

Appraisal: This report will be valuable in counseling the superior mathematics student. It is commendable that this report frequently mentions the importance of a broad background to supplement specialization in mathematics and statistics.

B. 162—The Evolution of the Calendar

World Calendar Association, 630 Fifth Avenue, New York 20, New York.

Booklet; 11 pages, 6″×9″; free.

Description: Here are described the various systems that have been used as bases for a calendar. As early as 4300 B.C. the Egyptians had a calendar year of 365 days. The true length of a year—365 days, 5 hours, 48 minutes, 46 seconds—has necessitated numerous changes in the calendar to keep it in step with the seasons. Major changes were made by the Julian calendar which made the year 46 B.C. 345 days long. The Gregorian calendar increased calendar accuracy by changing the leap year rule and dropping ten days from the year 1582. The story ends by showing how the World Calendar now being proposed would correct many of the irregularities of our present calendar.

Appraisal: The story of our calendar is well told in this brief article. It will be easily read by junior high school pupils.

B. 163—Can I Be An Engineer?

Department of Public Relations, General Motors, Detroit 2, Michigan.

Booklet; 6″×8″; 20 pages; free.

Description: It is the purpose of this booklet to encourage high school students to prepare to become engineers. To do this, it describes the importance of engineering, the different engineering fields, and the opportunity for employment in the future. Opportunities in engineering include the military services as well as private industry. The students who have the best prospects for success in engineering are those interested in mechanics, science, mathematics, in sketching ideas, and in visiting factories and exhibits. In high school the most important subjects for the prospective engineer are mathematics, science, English, shop, speech, economics and history.

Appraisal: This well-written booklet with attractive illustrations presents a strong case for engineering. Its specific suggestions with respect to aptitudes and subjects will be very helpful for counseling students. Mathematics teachers will be particularly pleased with the emphasis on mathematics as something more than just a required course. Since it is written to increase the supply of promising young people in engineering, it contains only two lines of advertising copy.

CHARTS

C. 43—Inventory by Airplane

United States Steel Corp., 71 Broadway, New York 6, New York.

Chart; 10″×16″; black and white; free.

Description: A photograph of a huge iron ore pile and a diagram are used to illustrate an ingenious method of determining the amount of iron ore or coal in an irregular pile. The method of measurement and computation used are explained in the written material accompanying the chart. The method involves aerial photography of the pile and fixed "datum"

points on the pile. Using a stereoscopic plotting machine a contour map is drawn from which the cubic volume can be computed. This method has reduced a ten week job to a matter of days with accuracy within 3 to 5 per cent.

Appraisal: This chart will be useful in showing how industry applies methematical principle in computing volume. It will be most appropriate for a solid geometry class.

C. 44—How Families Buy Life Insurance

Institute of Life Insurance, 488 Madison Ave., New York 22, New York.

Chart: 26″×36″; color; $0.15.

Description: This chart shows the steps to follow in buying life insurance to fit family needs. Using colorful drawings and captions, the chart shows how an insurance agent plans an insurance program that coordinates the assets and needs of a family group.

Appraisal: The material in this chart is presented in a colorful and concise fashion. Although no specific insurance company is advertised, the purpose of the chart is to sell the viewer the idea that life insurance is highly desirable for everyone. Teachers will find it useful in summarizing basic insurance information.

EQUIPMENT

E. 124—Fraction Fun

Miss Frances V. Clark, P.O. Box 3948, North Hollywood, California

Games; circles and fractional parts of circles; $.50.

Description: The cards for this game consist of two sets of the following fractional parts of a 5-inch circle: 1/1, 1/2, 1/3, 1/4, 1/6, 1/8, 1/9, 1/12. Each set of fractional parts contains enough sections to make a complete circle, for example, three thirds. The object of the game is to see who can complete the most circles of similar parts, that is, two halves, three thirds, four fourths, six sixths, eight eighths, nine ninths, or twelve twelfths.

Appraisal: The directions for this game show one way of making the comparison of fractions entertaining. The cards may also be used to illustrate a variety of processes such as addition, subtraction, and reduction. The cards are of good quality cardboard, brightly colored on one side and the fractional value printed on the gray side.

E. 125—Judy Counting Meter

The Judy Company, 310 North Second St., Minneapolis 1, Minnesota

Counting meter; 9″×16″; three 5-inch meters; $3.00 less school discount.

Description: This counting meter consists of three numbered dials connected by gears. Each dial is numbered with the digits 0 through 9. The right hand or unit gear is controlled by a dial similar to the dial on a telephone and can be rotated at will. When the unit dial makes one revolution the tens dial moves only one unit. Thus, when the number 9 of the unit gear is at the top, the next motion of the unit gear will cause the middle gear to move 1/10 of a turn; similarly when the tens dial makes one revolution, the hundreds dial moves only one unit. The dials are mounted on a board which can be placed in an upright position for class demonstration.

Appraisal: This device can be used to demonstrate the meaning of numbers by showing concretely the place value of a digit. It can also show how meters such as gas, water, electricity and speedometers work. It is of sturdy construction and painted in attractive colors.

E. 126—Arithmetic Dominoes

The Arithmetic Clinic, 4502 Stanford St., Chevy Chase, Maryland

Game; 104 cardboard dominoes; 7/8″×2″; $1.25.

Description: Each domino of this game has a number on the blue side and an addition fact on the reverse red side. The sum of the addition facts on each domino is the same as the number on the blue side.

The number on the blue side is given as a figure, as a series of dots and as a word. The addition fact on the red side is given in dots with the answer as a figure and a word. Directions are included for five games which give practice in counting, adding, writing numbers, reading number words, making number families, and in representing addition facts concretely and abstractly.

Appraisal: These dominoes will provide a variety of ways to present the basic addition facts from 0+0 to 9+9. The directions recommend groups of four players as being the most appropriate number. The direction sheet is very lengthy, is in small print, and intended to be read by an adult. Being of good quality cardboard, the game is an inexpensive way of building interest and emphasizing generalizations.

FILMSTRIPS

*FS. 162 to FS. 166—A Study of Measurement**

FS. 162—Part I—Linear Measure (40 frames)

FS. 163—Part II—The Story of Measurement (32 frames)

FS. 164—Part III—Lines, Angles, Surface Measurement (36 frames)

FS. 165—Part IV—Area of Rectangles, Area of Parallelograms (37 frames)

FS. 166—Part VIII—Special Measurement—Board Feet (Lumber Measure) Right Triangle (43 *frames*)

Photo and Sound Productions, 116 Natoma St., San Francisco 5, California

B & W ($4.00 each, $29.75 per set of eight).

Description of FS. 162: To illustrate the size of linear measures the inch is shown as approximately equal to the thickness of the thumb and the foot equal to 12 inches or one human foot. Drawings of rulers and yardsticks are used to show the subdivisions of units and how measurement is essentially a comparison process. A few illustrations are given to show that the unit of measurement to be used depends on the item being measured.

Description of FS. 163: Units of measure are shown to have developed from the use of parts of the human body for example the digit, hand, span, cubit, pace, foot, and yard. Other units such as acre or distance traveled were related to common experiences. Since these units varied according to individuals, it became necessary to standardize the units. Present day standards are illustrated.

Description of FS. 164—The meaning of lines, angles, and surfaces is shown and defined. The measurement of each of these by the process of comparison is illustrated using standard units.

Description of FS. 165: After illustrating the meaning of plane figures, rectangles, squares, perimeters, and area, the formula for computing the area of a rectangle is applied to a variety of rectangles. Similarly drawings show the relationship of a rectangle to an equal parallelogram and the area formula is applied to find the area of several parallelograms.

Description of FS. 166: The first half illustrates the meaning of board feet, and the computation of the number of board feet for several pieces of lumber. The second half is devoted to the Pythagorean theorem and its use in solving right triangles.

Appraisal: This filmstrip series can be used to show the meaning of different units of measure. It places emphasis on the concept of measurement as a process of comparison. However, more realistic applications should have been given instead of the simple geometric drawings. Instead of stating rules or formulas to be used, it would have been desirable to set up situations so that the viewer could have discussed and discovered the rule himself.

FS. 167—How Life Insurance Began (44 frames)

* Reviews of the other filmstrips in this series have appeared in previous issues as follows: Part V, *Area of Triangles—Area of Trapezoids*, FS. 73, Nov. 1950, p. 366; Part VI, *The Circle*, FS. 72, Oct. 1950, p. 286; Part VII, *Cubic Measure—Volume*, FS. 104, Feb. 1952, p. 126.

FS. 168—How Life Insurance Operates (41 frames)
FS. 169—How Life Insurance Policies Work (42 frames)
FS. 170—Planning Family Life Insurance (46 frames)

Institute of Life Insurance, 488 Madison Ave., New York 22, New York

B & W ($3.00 each, $10 for set) or Color ($5.00 each, $15 for set); rental free to schools; teacher's guide.

Description of FS. 167: This filmstrip briefly traces the history of insurance from its beginning in the sharing of shipping risks by Chinese merchants 5000 years ago to modern insurance policies. Illustrations are given to show the risks that man faces such as losses from fire, dangers of the sea, death, or old age. Early insurance plans began with marine insurance which even included ransom insurance for payments to pirates while plans for death payments were based on mutual organizations that shared equally the premiums paid by its members. Later mortality tables made insurance payments predictable so that insurance companies could expand rapidly.

Description of FS. 168: The loss of a textbook results in the establishment of a lost book fund by which students share the cost of losing textbooks. This is an introduction to an explanation of how the premiums of policy holders are used to pay death benefits. The establishment of premium rates is illustrated by an imaginary group of 10,000 policy holders. The rate of mortality of this group according to age is related to premium payments to show the advantage of the level premium system.

Description of FS. 169: It is the purpose of this filmstrip to show how different types of life insurance provide for varying needs. Term insurance may be the best protection for a man buying a house. Straight life insurance is most suitable for the man who needs life-long protection at the lowest rate. Limited payment life insurance is most appropriate for the man who wishes to pay for his insurance when his income is high. The endowment insurance is illustrated as being the best for a career girl to meet emergency needs or provide retirement benefits.

Description of FS. 170: This filmstrip emphasizes the role of the insurance agent in planning an insurance program to meet the specific needs of his clients. It is the job of the agent to determine the protection needed by the family such as the payment of the home mortgage, a life income for the surviving wife, or an emergency fund to meet family needs. Illustrations are given on life benefits such as retirement payments if the policy holder does not die. Suggestions are given for continuous planning of the insurance program of a family to meet new needs or to coordinate insurance policies with participation in other related programs such as social security.

Appraisal of FS. 167 to FS. 170: These filmstrips use colorful drawings to illustrate vividly the function of life insurance. The illustrations are simple and direct in a manner that should make the topics readily understood by the viewer. Since it is the purpose of these filmstrips to sell the idea of life insurance to meet all needs, no mention is made of other ways of meeting these needs. Another omission is the limitation of income in the purchase of insurance. The accompanying teacher's guide includes suggested uses, content, activities, quiz and references that should make it possible for the teacher to use the filmstrips effectively. The amount of specific mathematical content is limited.

INSTRUMENTS

I. 39—Number-ite

The Judy Company, 310 North 2nd St., Minneapolis 1, Minnesota.

Number boards; ten 3″×5½″ boards; 60 pegs; $2.50 to schools.

Description: These ten number boards each have one of the numbers from 1 to 10 painted on its surface. Each board has as

many holes as the number painted on the board. Colored pegs fit into these holes to show the grouping, which match the number symbol. Each board has notches according to its number so that the boards fit together only when the number sequence is consecutive.

Appraisal: This is a simple, inexpensive device for relating a number symbol to its quantitative value. The boards are well made out of masonite with a wood veneer surface die cut.

Applications

(*Continued from page 367*)

From this equation we may obtain explicit expressions for I and for n. Directly, we find

$$I=\frac{iB_0}{1-(1+i)^{-n}}. \qquad (3)$$

After taking logarithms

$$n=\frac{\log I-\log (I-iB_0)}{\log (1+i)}. \qquad (4)$$

Formula (3) is useful in determining a tentative value of I for a specified whole number n of time periods. A conveniently "rounded" value of I afterward being agreed upon, formula (4) gives the exact n. This is almost certainly not an integer. Hence the final payment is less than I. The number of time periods is then the smallest integer greater than n.

In an actual case a \$6500 loan earning 5% per annum was to be retired in ten years by monthly payments. Putting $n=120$, $i=5/1200$, $1+i=1205/1200$ in (3) gave $I=68.942$. The borrower insisted on no more than 68.95. This in (4) made $n=119.9826$. Thus there were to be 120 payments, the last one \$67.75, each of the others \$68.95. After 42 payments a refinancing became desirable. Equation (1) shows $B_{42}=\$4582.70$. the principal for the new situation.

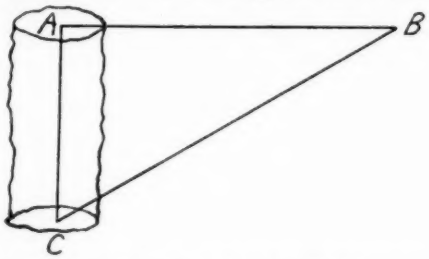
It may be noted that the Financial Publishing Company of Boston, Massachusetts, furnishes at small cost a table called a "Loan Amortization Schedule" for any specified amount, interest rate, and either number of installments or installment amount. This is usually a single sheet which lists the interest-paying part, the debt-reducing part and the unpaid balance for each and every payment required to clear the debt. Besides its obvious value to borrower and lender alike, such a table is useful and interesting to a class studying these matters.

For a graphical treatment of the topic here considered, interested readers may consult pp. 39–42 of the February 1935 issue of the *American Architect* magazine.

T. 6 Gr. 8–12 *The Tangent Function Saves a Life*

JOHN KINSELLA of New York University sent us the following.

A very real, human-tragic application of the tangent function occurred in Brooklyn last year. A man was trapped in a cave-in.



Digging close to the hole would have endangered him by further cave-ins. It was necessary to dig obliquely along a line BC from point B. In order to provide digging directions, it was necessary to determine angle ABC, given $AC=35$ feet, $AB=50$ feet, and $AC\perp AB$.

All New York was on edge. One of my student teachers had his class at maximum interest as he dramatically discussed the situation. You could hear a pin drop! This was not functional mathematics in a bread-and-butter sense; this was mathematics in a life and death sense.

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

High School

Plane Geometry, (Revised ed.), by Rachel P. Keniston and Jean Tully. Cloth, vii+392 pages, 1953. Ginn and Co., Statler Building, Boston 17, Mass. $2.88.

Plane Geometry, by Virgil S. Mallory and Chauncey W. Oakley. Cloth, x+468 pages, 1953. Benj. H. Sanborn and Co., 221 E. 20th St., Chicago 16, Ill. $2.64.

Plane Geometry, A Clear Thinking Approach, (Third ed.), by Leroy H. Schnell and Mildred G. Crawford. Cloth, xii+436 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $3.20.

Basic Ideas on Mathematics, by Francis G Lankford, Jr., and John R. Clark. Cloth, viii+504 pages, 1953. World Book Company, Yonkers-on-Hudson, N. Y. $2.84.

College

College Algebra and Plane Trigonometry, by James H. Zant, Oklahoma Agricultural and Mechanical College. Cloth, x+337+82 pages, 1953. Ginn and Co., Statler Building, Boston 17, Mass. $4.00.

Algebra for College Students, by Ross R. Middlemiss, Washington University. Cloth, viii+394 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $3.75.

Elements of Algebra for Colleges, by V. B. Caris, Ohio State University. Cloth, vi+307 pages, 1953. Ginn and Co., Statler Building, Boston 17, Mass. $3.30.

Algebra (2 Vols., Sixth ed.), by G. Chrystal, University of Edinburgh. Cloth, xxiv+571 pages, xxiii+616 pages, 1952. Chelsea Publishing Co., P.O. Box 55, Washington Bridge Sta., New York 33, N. Y. $8.00.

Fundamental Concepts of Geometry, by Bruce E. Meserve, University of Illinois. Paper, v+296 pages, 1953. Addison-Wesley Publishing Co., Kendall Sq., Cambridge 42, Mass.

Plane Trigonometry with Tables, (Second ed.), by Donald H. Ballou, Middlebury College; and Frederick H. Steen, Allegheny College. Cloth, vi+195 pages+answers and tables, 1953. Ginn and Co., Statler Building, Boston 17, Mass. $3.25.

Plane and Spherical Trigonometry with Tables (Second ed.), by Donald H. Ballou, Middlebury College and Frederick H. Steen, Allegheny College. Cloth, vi+205 pages+answers and tables, 1953. Ginn and Co., Statler Building, Boston 17, Mass. $3.50.

Differential and Integral Calculus, by Philip Franklin, Massachusetts Institute of Technology. Cloth, xi+641 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $6.00.

Analytic Geometry and Calculus, by Lloyd L. Smail, Lehigh University. Cloth, xiv+644+lxx pages, 1953. Appleton-Century-Crofts, Inc., 35 W. 32nd St., New York 1, N. Y. $5.50.

An Introduction to Statistics, by Charles E. Clark, Emory University. Cloth, x+266 pages, 1953. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $4.25.

An Introduction to Plane Projective Geometry, by E. J. Hopkins and J. S. Hails. Cloth, vi+276 pages, 1953. Oxford University Press, 114 Fifth Avenue, New York 11, N. Y. $5.50.

Theory and Applications of Distance Geometry, by Leonard M. Blumenthal, University of Missouri. Cloth, xi+347 pages, 1953. Oxford University Press, 114 Fifth Ave., New York 11, N. Y. $10.00.

Complex Analysis, by Lars V. Ahlfors, Harvard Uinversity. Cloth, xi+247 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $5.00.

Geometry and the Imagination, by D. Hilbert and S. Cohn-Vossen. (Translated by P. Nemenyi), Cloth, ix+357 pages, 1952. Chelsea Publishing Co., P.O. Box 55, Washington Bridge Station, New York 33, N. Y. $5.00.

Numerical Solution of Differential Equations, by William Edmund Milne, Oregon State College. Cloth, xi+275 pages, 1953. John Wiley and Sons, 440 Fourth Ave., New York 16, N. Y. $6.50.

Introduction to Measure and Integration, by M. E. Munroe, University of Illinois. Cloth, x+310 pages, 1953. Addison-Wesley Publishing Co., Cambridge 42, Mass. $7.50.

Logic for Mathematicians, by J. Barkley Rosser, Cornell University. Cloth, xiv+530 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $10.00.

Demand Analysis, A Study in Econometrics, by Herman Wold, University of Uppsala, and Lars Jureen, State Agricultural Marketing Board of Sweden. Cloth, xvi+358 pages, 1953. John Wiley and Sons, 440 Fourth Ave., New York 16, N. Y. $7.00.

Beginning Algebra for College Students, (Second ed.), by Lloyd L. Lowenstein, Kent State University. Cloth, xiii+279 pages, 1953. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $3.50.

Calculus and Analytic Geometry (Second ed.), by George B. Thomas, Jr., Massachusetts In-

stitute of Technology. Cloth, xi+731 pages, 1953. Addison-Wesley Press, Inc., Kendall Square, Cambridge 42, Mass. $8.50.

Introduction to the Theory of Statistics, by Victor Goedicke, Ohio University. Cloth, xii +286 pages, 1953. Harper and Brothers, 49 East 33rd St., New York 16, N. Y. $4.50.

Sampling Techniques, by William G. Cochran, The Johns Hopkins University. Cloth, xiv+330 pages, 1953. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $6.50.

Fundamentals of College Mathematics, by Richard E. Johnson, Smith College; Neal H. McCoy, Smith College; and Ann F. O'Neill, Wheaton College. Cloth, xiv+479 pages, 1953. Rinehart and Company, 232 Madison Ave., New York 16, N. Y. $6.00.

Real Functions, by Casper Goffman, Wayne University. Cloth, xii+263 pages, 1953. Rinehart and Company, 232 Madison Ave., New York 16, N. Y. $6.00.

Teaching of Mathematics

The Teaching of Secondary Mathematics, by Claude H. Brown, Central Missouri State College. Cloth, xi+388 pages, 1953. Harper and Brothers, 49 E. 33rd St., New York 16, N. Y. $4.00.

Miscellaneous

Flatland, A Romance of Many Dimensions, (Sixth ed., Rev.), by Edwin A. Abbott. Paper, 109 pages, 1952. Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $1.00 (clothbound, $2.25.)

Introduction to Logic, by Irving M. Copi, University of Michigan. Cloth, xvi+472 pages, 1953. Macmillan Co., 60 Fifth Ave., New York 11, N. Y. $4.00.

Youth—The Nation's Richest Resource, Their Education and Employment Needs, by The Interdepartmental Committee on Children and Youth. Paper, x+54 pages, 1951. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.20.

BOOK REVIEWS

Working With Numbers, Book 7, James T. Shea. Austin, Texas, The Steck Company, Publishers, 1952. Paper, 128 pp., $0.56.

This worktext series is unique in many important respects. In Book 7, the first pages include review pages in all the operations, perimeters, areas, volume, "The Vocabulary and Symbols of Arithmetic," "Indicating the Processes," and "Mental Shortcuts." As percentage and other topics are taken up the advantages of the text book and the work book are combined by actually developing and explaining the various new arithmetical processes used on each page before asking the pupil to use them in problem solving. Then follows good illustrative problems. Each new process and each type of difficulty is presented separately and with much explanation.

Development of right thinking and correct approach are stressed. The arrangement of the printed matter and space left for work is conducive to neatness and orderliness of written work.

The subject matter of the seventh grade is well covered in a pleasing, teachable and learnable manner.—Mrs. Lorena Holder, John H. Reagan School, Dallas, Texas.

Growth in Arithmetic Grade Eight, John R. Clark, Rolland R. Smith, and Harold E. Moser. Yonkers-On-Hudson, New York, World Book Company, 1952. v+314 pp., $2.12.

This book is the sixth in a series starting with the book for grade three. All of the books use attractive illustrations in color. Each page is inviting and easy to read. The book covers the usual topics expected in a book for grade eight. The geometry coverage is generous, ranging from the geometry found in design through work with protractor, compasses and ruler to the use of congruent and similar triangles in indirect measurement. In addition to the usual work with formulas, the algebra work includes about six pages of work with simple equations. The explanation of solutions of equations was made without the use of axioms. Most algebra teachers would consider this a serious omission.

The first half of the book develops skill in estimation. The pupil is encouraged to work independently through such devices as *Self-Help Tests, Be Your Own Teacher, Holding Your Ground* (oral and written review) which appear regularly throughout the book.—Elizabeth J. Roudebush, Director of Mathematics, Seattle Public Schools, Seattle, Washington.

Arithmetic in Life and Work (Fourth ed.), Sidney J. Lasley and Myrtle F. Mudd. New York, Prentice-Hall, Inc., 1952. x+259 pp., $1.96.

This book designed for ninth grade or a refresher course places emphasis on basic arithmetic. It outlines the mathematical processes in a very concise and systematic manner and gives abundance of material for practice in both computation and verbal problem situations. There are numerous inventory tests with individual follow-up practice exercises designed to overcome indicated weaknesses. The many review exercises supplemented by achievement tests should provide ample material for checking maintenance and improvement of skill. The book offers many topics which deal with the problems of the consumer.

The material although quite readable has a helpful glossary that should be of real aid to those who have reading difficulties. The optional exercises and the suggested activities should serve to enrich the course for the more gifted pupils. The format of the book is very pleasing and the pictures used are not only attractive but functional.

This book would serve as a useful tool in the hands of a skillful teacher who could help pupils: to see reasons for and the inter-relation of procedures; to analyze and to estimate so that greater independence in working with number could be developed; to generalize so that the individual might have greater opportunity to develop deeper insight as to the function of number. Although the book is well organized it does not give a great deal of help in this direction.—Olive Wear, Fort Wayne Public Schools, Fort Wayne, Indiana.

General Mathematics at Work, Claude H. Ewing and Walter W. Hart. Boston, D. C. Heath and Co., 1952. v+266 pp., $2.80.

Much of the material in this textbook was originally used in the book entitled *Essential Vocational Mathematics* (1945). Just as that book presented a one year course in general mathematics centered about the need of workers in the fields of construction and industry, so does this one. Twenty-two different fields of industrial employment were surveyed to determine the choice of material to be presented.

The text covers the field of mathematics from a review of arithmetic to the trigonometry of the right triangle. The chapter on precision measuring tools is very complete and good.

The diagrams in the entire book are very well done but a little more interest might have been aroused if some more photographs had been added.

One feature which was particularly attractive to the reviewer was the amount of drill material given both on the arithmetic level and on the new work covered in the course.

Even though a teacher were not teaching a course in vocational mathematics this book would make a valuable addition to the classroom library since it covers much material not generally found in one book.—Madeline D. Messner, Abraham Clark High School, Roselle, New Jersey.

Mathematics In Action, Bk. I (Third ed.), Walter W. Hart and Lora D. Jahn. Boston, D. C. Heath and Company, 1952. xi+324 pp., $2.12.

Mathematics in Action, Bk. I is a practical course in mathematics carefully designed for seventh grade. Containing the usual topics for seventh grade, it supplies a maximum number of problems, definitions, and exercises in the fundamentals. Worthwhile added features are progress tests and cumulative reviews. Although the print is smaller than some texts and explanatory material is brief, the directions are clear and concise. Optional material is provided. This book should be an asset to the teacher teaching individuals.—Kenneth R. Fish, Wisconsin State College, La Crosse, Wisconsin.

General Trade Mathematics (Second ed.), Edwin P. Van Leuven. New York, McGraw-Hill Book Company, Inc., 1952. ix+553 pp.-$3.80.

This edition of the author's 1942 text offers little in the way of new material but, like the first edition, is very well done and quite complete as a teaching text or reference work for the field of basic shop mathematics. As might be expected, in a one volume text, there is nothing offered in trigonometric applications, compound angles, gear design, or other advanced topics.

It is difficult to justify, in the mind of the reviewer, the omission of material on statistical and graphical interpretation which was contained in the earlier edition.

Mechanical rules are clear, printed in bold type, and have adequate examples. There is little emphasis on "why" but much on "how."

The context has been brought up-to-date on matters of price and wage changes. The chapter on "Vocational Finance" is unusually well done in language understandable to the average student or worker, discussing group insurance, taxes, Social Security, budgeting, compensation, life insurance, and related topics.

Quantity and quality of problems, topic arrangement, illustrations, indexing, appendix (tables), and general form are excellent.

The book could be used effectively as (1) A text in industrial or trade schools, (2) Reference work for a home library or an industrial worker, (3) Reference or text for selected topics in the high school shop courses, (4) Source of practical and enriching problems in high school algebra, geometry, or even business education.—R. V. Belding, Thomas Carr Howe High School, Indianapolis, Indiana.

Dynamic Solid Geometry, David Skolnik and Miles C. Hartley. New York, D. Van Nostrand Co., 1952. viii+231 pp., $2.48.

The physical features make this book attractive at the very first glance. The book is neat and compact and has clear print and an abundance of excellent illustrations. A review, an adequate test, and problems for superior students are at the end of each of the five units in the book. The course is so flexible as to permit three different forms of presentation: (1) logical structure and applications receive about equal attention; (2) original and numerical problems receive more stress than propositions; (3) a course of less than one semester with the whole stress on numerical exercises. An outline of the short course is given in the back of the book.—Winnie Macon, Haskell Institute, Lawrence, Kansas.

Zahl und Raum, (4 Vols.) by Fritz Malsch. Heidelberg, Verlag Quelle und Meyer G.M.B.H.; Offenburg/Baden, Lehrmittel-Verlag G.M.B.H., 1949. I Teil, Geometrie, 127 pp., DM 2.80; II Teil, Arithmetik und Algebra, 192 pp., DM 2.80; III Teil, Arithmetik, Algebra, Analysis, Stereometrie, 111 pp., DM 2.60; IV Teil, Trigonometrie, Analytische Geometrie, 111 pp., DM 2.60.

These four small volumes are a revision of texts written by Malsch, Maey (Dr. Eugen) and

Schwerdt (Hans) in 1925–27. The amount of material in the texts has been cut because there is too much "stuff" in the mathematics courses and cost of publication and availability of material make a smaller book necessary. The texts are planned to provide the needed material for a variety of interests, from those related to the humanities (which usually interest a minimum number of students in mathematics), to those finding application in the technology. The material included must meet the mathematical needs of citizenship, permit the pupil to realize the true spirit of the subject as the science of number and space, and use applications to science and industry.

The author hopes that the pupil will realize that the laws of order of number and space which man's intelligence has invented are the laws which also govern the universe. He claims that the book is written so that the pupil may make up any work he has missed without the teacher's aid.

The topics considered in each volume are as follows: Part I, Geometry: symmetry, straight lines, triangles, quadrilaterals, circles, similar figures, regular polygons, cross sections of solids. Work on locus, inequalities, construction, variation of given conditions, computation are all treated within each unit. Part II, Arithmetic and Algebra: an introduction to the general number, computation of algebraic quantities, signed numbers, fractions, graphing, liner equations, powers and roots, quadratic equations, logarithms and exponential theory, slide rule and calculating machines. Part III, Arithmetic, Algebra, Analysis, Stereometry: progressions (computation as well as geometric construction), compound interest, binomial theorem for positive, integral exponents, the number system including complex numbers, theory of equations, differentiation of rational functions, maxima and minima, trigonometric functions, infinite series, introduction to integrals, stereometry, spherical geometry. Part IV, Trigonometry, Analytic Geometry: plane and spherical trigonometry, including some mathematical astronomy (pages 1–57), the coordinate system, lines and circles, conic sections (theory and construction).

Each volume contains a page or more of paragraphs on the life of each mathematician mentioned in that volume, a table of formulas developed and used in the text, a table of contents and index, and a glossary of mathematical terms used.

In reading these books, considering the presentation, application and integration of subject matter, one wonders whether the time has come for a rethinking of the vast amount of "stuff" found in high school texts used in the United States.—MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois.

What Does Research Say About Arithmetic, Vincent J. Glennon and C. W. Hunnicut. Washington, D. C., Association for Supervision and Curriculum Development, 1952. Paper, iv+45 pp., $0.50.

The purpose of this bulletin is to present to the teacher and other school personnel as well as to the interested parent a summary of the theoretical and scientific knowledge of the place of arithmetic in the modern elementary school. An examination of the bibliography of one hundred twenty-five items indicates the extent of the research which has been used. Because the drill theory has been shown to be invalid as a learning theory, earlier research studies have lost most of their value and the studies reported are largely from those of the past fifteen years.

The questions covered were suggested by teachers and supervisors and their answers checked by professional personnel on the elementary school level. The thirty-eight questions range from those of broad general interest concerning the importance and methods of developing meaningful arithmetic to such specifics as, "Should we teach children to use crutches?" and "Does homework help?"

Someone has said that there is a lag of fifty years between the introduction of a theory in education and its practices in the classroom. Such concise, easily read pamphlets as this one should shorten that time. School personnel will find in it a scientific answer to the frequent criticism by the public of arithmetic results.

Teachers and others interested in the elementary curriculum will find here in one small volume suggestions for improvement based upon research, as well as a rich bibliography for further study. The ASCD Executive Committee and the Research Board are to be commended for the publication of such a book. *No elementary school can afford to be without one.*—LESTA HOEL, Portland Public Schools, Portland, Oregon.

Didaktik und Methodik des Mathematischen Unterrichts, Kuno Fladt. Frankfurt, Hirschgraben-Verlag, 1950. 168 pp.

This stimulating and scholarly book deserves the careful attention of all who are concerned with the teaching of mathematics and the education of mathematics teachers. The author, well-known in educational circles in Germany, has crystallized the pedagogical experience of forty years in this slender volume. Two important keynotes are sounded early: (1) mathematics can never be learned by using the Nurnberg funnel—mathematical understanding can be achieved only through slow, steady growth; and (2) subject matter content exhibits a remarkable invariance over the years despite endless variations in curriculum organization.

The first three chapters are devoted to mathematics and education—intuition, mental discipline, concentration and rigor, the relation of pure mathematics to applied mathematics, the relation of mathematics to science, and the humanistic and cultural bearings of mathematics. In short, a cultured person is one who has a genuine understanding and appreciation of the highest intellectual heritage of man—and

mathematics can no more be omitted in this connection than the humanities, the sciences, or the fine arts.

The next nine chapters, about half of the book, are concerned with specific mathematical content (topics, concepts, etc.) presented systematically by grade levels, from the fundamentals of arithmetic and intuitive geometry to spherical trigonometry, coördinate geometry and calculus. There is also a schematic summary of topics showing grade placement. The accompanying motto is that "syllabi are never as complicated as they would appear to be." Significantly, there is here indicated an unusually thorough correlation, in the best sense, between the various branches of mathematics—arithmetic, algebra, elementary geometry, elementary analysis, trigonometry, descriptive geometry, analytical geometry, calculus and astronomy.

The last four chapters deal with methodology in general and historical terms, reviewing past trends and the influence of such leaders as Felix Klein, A. Höfler, W. Lietzmann, Ph. Maennchen, Max Simon, J. Henrici, P. Treutlein, W. Killing and H. Hovestadt. The last dozen pages, devoted to observations on the relation of mathematics to other school subjects, are to be commended.—William L. Schaaf, Brooklyn College, Brooklyn, New York.

Plane Trigonometry (Third ed.), Fred W. Sparks and Paul K. Rees. New York, Prentice-Hall, Inc., 1952. viii+275 pp., $3.50.

The content of the third edition of this text does not differ appreciably from that of the revised edition. The general trigonometric functions are introduced immediately, while solutions of right triangles (non-logarithmic) are introduced early enough (Chapter 2) to give the student a knowledge of the practicality of the subject. The elementary identities are prefaced with fine suggestions for attacking type problems. Although the authors make the distinction between conditional equations and identities, there is no set of exercises asking the student to differentiate between them.

The book contains a large number of well-selected exercises which are well ordered as to difficulty. Its coverage is excellent, with four appendices as follows: a) The Versed Functions, b) Mollweide's Equations, c) Logarithms to Bases other than Ten, and d) An Extension of the Proof of the Addition Formulas. The explanations are clear and complete, yet not wordy. It makes the text very readable and hence teachable.—George Bullis, Wisconsin State College, Platteville, Wisconsin.

General College Mathematics, W. L. Ayres, Cleota G. Fry, and H. S. F. Jonah. New York, McGraw-Hill Book Company, Inc., 1952. xii+283 pp., $3.75.

This book is designed to provide a one-year course in terminal general education mathematics for first year college students who do not intend to specialize in mathematics or closely related fields. This very well written and extremely interesting text assumes a high level of mathematical maturity on the part of the students. Topics include algebra, trigonometry, statistics and probability, and "glimpses" of modern mathematics such as topology, theory of numbers, the basis of our number system, and logic. Unusual mathematical recreations are included in each chapter under the heading of "problems just for fun." Historical backgrounds and famous mathematicians are appropriately introduced. Development of concepts is precise and adequately thorough. The book can be criticized on the minor points of minimum number of problems and omission of answers to the recreational problems. Consideration of this book by college instructors interested in general mathematics is a must.—Arthur J. Hall, San Francisco State College, San Francisco, California.

Revision Course in General Mathematics, Clement V. Durell. London, G. Bell and Sons Ltd. 1952. vi+125 pp., 6 shillings.

The purpose of *Revision Course in General Mathematics* is to provide a "revision" or review course in mathematics for students who take the examination for the General Certificate of Education in England.

The subject matter is arranged by mathematical topics. These topics are arithmetic, algebra, geometry, and trigonometry. Four trigonometric tables and answers are included.

Since there are only one hundred and twenty-five pages of content exclusive of the tables, and since content is presented as review, the explanatory material is brief and is usually confined to definitions, formulas, and illustrative examples.

Most of the content consists of exercises and problems within the topics named.

There are some differences in the use of symbolism as compared with customary practice in the United States. For example, the money notation follows English practice. Also, the decimal point is placed in the center of the vertical line space rather than at the bottom.

Since explanations are lacking, and since the exercises are quite sketchy in their content coverage, it seems that the book would not be of great value to students who are not already familiar with the mathematics involved.

However, there is a wide variety of practice material, and a wide variety of problem applications contained in the book.

This book should be valuable as resource material for high school mathematics teachers.—Dale Carpenter, Los Angeles City Schools, Los Angeles, California.

Basic Mathematics for Technical Courses (Second ed.), Clarence E. Tuites. New York, Prentice-Hall, Inc., 1952. x+438 pp., $3.75.

This is a revision of a text for "technical institutes and the junior college field." It is a systematic and workmanlike job covering those

parts of arithmetic, algebra, and trigonometry considered to be of specific value to the technician. Many of the problems are drawn from a variety of fields where mathematics is used.

The character of the book is indicated by the unusual features it possesses. Among these are the replacement of the mathematician's imaginary unit i by the j of the physicist and the engineer; the presentation of certain problems in the form met in mechanical drawing; and the emphasis on the use of the slide rule, which is taught in Chapter I as pure manipulations whose explanations are deferred for 228 pages.

The new edition extends the work in determinants to include solution of systems of equations with more than three unknowns; adds the topic of synthetic division and substitution; and discusses polar vectors as well as rectangular.—RALPH C. HUFFER, Beloit College, Beloit, Wisconsin.

Mathematics For Finance and Accounting, J. B. Coleman and William O. Rogers. New York, Pitman Publishing Company, 1949. x+310 pp., $4.00.

This text is designed for an introductory course for students interested in this field. The mathematics background required is the ability to solve equations, and an understanding of negative and fractional exponents. Logarithms and computation with approximate data, essential to the computations, are treated concisely but adequately.

In the first seven chapters (112 pages), the basic formulas and understandings are presented. The remaining seven chapters treat (1) applications to sinking funds, amortizations, bonds, depreciation, perpetuities, and capitalized costs, and (2) an introductory treatment of life insurance.

The format of the book is excellent—good paper, large readable print, and good spacing. Explanatory treatments are short but clear. A wealth of exercises is found after each topic; answers are provided for odd numbered problems. The tables are complete, differing from the usual tables in the use of $1/s\overline{|}_i$ instead of $1/a\overline{|}_i$ and the use of the Commissioners Standard Ordinary Mortality Table (1941) with commutation columns at 2%. The authors present a useful method for finding an upper bound to the errors of linear interpolation.

Instructors should find this a very teachable text and that it compares favorably with other texts in this field.—LESTER R. VAN DEVENTER, Eastern Illinois State College, Charleston, Illinois.

Advanced Mathematics for Technical Students, Part 2, H. V. Lowry and H. A. Hayden. New York, Longmans, Green and Co., 1951. ix +422 pp., $3.75.

This textbook covers the material usually presented to engineering students in the third or fourth year of college. The selection of material is excellent and the presentation is logical. The proofs are rigorous.

Especially interesting is the treatment of the convergence of integrals, Fourier Series and solid analytic geometry. The chapter on differential equations is adequate.—HERBERT L. LEE, University of Tennessee, Knoxville, Tennessee.

Three Pearls of Number Theory, A. Y. Khinchin. Rochester, N. Y., Graylock Press, 1952. 64 pp., $2.00.

This unusual little book, judging from the author's prefatory remarks, is essentially a "long letter" written to one of his students serving in the Soviet (this book is an English translation of a Russian edition) armed forces during the last war. It was written in answer to a request for some theorems in number theory to study. The work is truly inspiring and is written with great understanding. The author goes to considerable effort to explain each step in the proofs—in order that they may be understood by a person of modest mathematical attainment.

Three main theorems are discussed and proved. The first is van der Waerden's theorem on arithmetic progression. The second chapter is concerned with the density problems studied by Schnirelmann and gives the Artin-Scherk proof of the α, β hypothesis. The third chapter is devoted to Linnik's solution of Waring's problem.

The proofs given are elementary in the technical sense but are nevertheless quite complicated. This is largely mitigated by the excellent exposition. Hence the study of the proofs will be richly rewarding to all who take the trouble to work through them. It would be a splendid idea to have this book available to young students, even at senior high school level, since it would afford great stimulation to prospective mathematicians.—W. E. JENNER, Northwestern University, Evanston, Illinois.

Algebraic Projective Geometry, J. G. Semple and G. T. Kneebone. Oxford, Oxford University Press, 1952. vii+404 pp., $7.00.

This book presents a very detailed exposition of the projective geometry of the line, plane and three space over the complex numbers. Algebraic methods are used throughout in preference to synthetic ones, although the underlying geometrical ideas are brought out as much as possible.

In the first chapter the authors present a rather informal introduction to abstract axiom systems; the ideas of a field, ring and group are introduced and Klein's Erlanger Program is discussed. It is unfortunate that although fields and groups are used in the discussion, no rigorous definition of these concepts appears in the book. After devoting a chapter to an intuitive account of projective, affine and Euclidian geometry and the relations between them, the whole subject is developed rigorously. The methods of linear algebra are used throughout and a knowledge of elementary matrix theory is essential to the

reader. The ground field is usually the complex numbers although specializations to the reals occur frequently. Affine and Euclidian theorems are obtained from projective ones by selecting a plane at infinity and choosing an absolute conic therein. The derivation of non-Euclidian geometries from projective geometry is sketched at the appropriate places in the text but not discussed in detail.

The usual topics: collineations and correlations, conics, quadrics and twisted cubics are treated in great detail, but there is also a brief treatment of such topics as quadratic transformations, invariants and cubic surfaces. The last chapter of the book contains a brief account of the geometry of four and five dimensional space. Many excellent exercises occur throughout the text.—Alex Rosenberg, Northwestern University, Evanston, Illinois.

Calculus of Variations with Applications to Physics and Engineering, Robert Weinstock. New York, McGraw-Hill Book Co., 1952. x+326 pp., $6.50.

This book offers an introduction to the calculus of variations and a survey of the applications of the calculus of variations to problems in physics and engineering. It presupposes nothing more than the average course in advanced calculus and a little ordinary differential equations.

The author develops the Euler-Lagrange equations and then proceeds with the applications. He does not discuss any of the sufficiency criteria of the existence problems, which helps to make the book more readable for the engineer.

Some of the applications are to geometrical optics, dynamics of particles, vibrating strings and membranes, elasticity, quantum mechanics, and electronics. These subjects are introduced in such a way that no previous familiarity with them is required for complete understanding.

Each chapter is appended with a list of exercises for the student. Here are also references to other works to supplement the material covered. —G. Springer, Northwestern University, Evanston, Illinois.

Analytic Geometry, W. K. Morrill. Scranton, Pennsylvania, International Textbook Co., 1951. xii+383 pp., $3.50.

A course in analytic geometry based on this text will adequately study functional behavior and will coordinate notation and methods to move the student easily into the calculus and subsequent courses in mathematics.

Early introduction of the concept of increment is a blessing; translation of axes might come sooner. Problems of loci occurring many places in the text is good; the instructor must, however, be alert to the need for emphasizing graphing to develop students' ideas of functional behavior. All texts seem to include artificial problems like Example 6–2, page 175. It is refreshing to see "not defined" used in tables of trigonometric functions instead of the symbol, ∞; and "*s*" is used for slope in the place of the overworked "*m*." Stating general conditions, *any* fixed line and *any* fixed point to develop either the general conic or the conic in standard position, then placing axes conveniently would improve the logic in finding the equations of the loci.

Likely to set a trend in courses and textbooks to come is the emphasis on vector methods coloring the treatment of slope of a straight line, direction cosines of a line, parametric equations and rotation of axes by direction cosines.—Leota C. Hayward, Colorado A and M College, Fort Collins, Colorado.

Elementary Coordinate Geometry, E. A. Maxwell. London, Oxford University Press, 1952. 288 pp., $3.50.

American teachers will find this text by an English author interesting reading. The nature of the materials presented is indicated by the table of contents which lists the following twenty chapters: Coordinates, The Straight Line, Digression on Linear Equations, Elimination and Determinants, The Straight Line (continued), The Straight Line in "Perpendicular" Form, Introduction to Analytical Methods, Two Standard Curves, The Circle, The Ellipse, The Hyperbola, The Use of Calculus, Some Typical Curves, Envelopes, Focus and Directrix, Some Problems on Tangency for Conics, Geometrical Properties of the Parabola, Geometrical Properties of the Ellipse, Geometrical Properties of the Hyperbola, and Polar Coordinates. The author suggests various other sequences that may be followed.

The average class might find parts of the text a little difficult to read and to keep organized. In the first chapter, the author has introduced such topics as translation and rotation of axes, theorem of Menelaus, parameter equations (which are not defined until fifty pages later). The author considers determinants and the calculus as fundamental problems in equipment, but he does not insist on their use.

The author uses parametric equation definitions for his introductory work with all the conics except the circle. For the circle he first uses a locus definition and then later introduces the parametric one. The latter portion of the work gives a detailed study of the conics by using closely interwoven methods of pure and analytic geometry. Polar coordinates are mentioned on page twelve and explained briefly in the concluding chapter. No work on solids is included.

A number of unusual problems should be invigorating to superior students. The book should prove to be a valuable reference for students and for teachers.—Francis R. Brown, Iloinois State Normal University, Normal, Illinois.

Affiliated Group Activities

By JACKSON B. ADKINS
Committee on Affiliated Groups
Phillips Exeter Academy, Exeter, New Hampshire

PROGRAM IDEAS FROM AFFILIATED GROUPS

The Committee on Affiliated Groups of the National Council of Teachers of Mathematics has accumulated some information and ideas on the making of programs. We pass it on to readers of THE MATHEMATICS TEACHER in the hope that it may be of some help in that never-ending, knotty problem of building a worthwhile and attractive program for an organization.

Of course, the larger the group and the longer the meeting, the easier it is to plan the program. With a large group, meeting for several days, it is possible to provide a variety sufficient to meet all tastes and needs and it is also possible to achieve a continuity that is frequently impossible in short meetings. Planning four or five two-hour programs a year for a group whose average attendance may run from thirty to eighty is the really difficult job. Without further generalizations we give you some of the ideas that have come along to us.

The joint meeting of the National Council and The New England Institute produced several new ideas. One of these was a study group entitled "The Influence of Group Dynamics on Teaching and Learning." This is a topic that is of interest to teachers of all levels and might provide a series of meetings that could be quite exciting and worthwhile. The outline for the Institute group was

1. What is group dynamics?
2. Analyzing the behavior of a group.
3. Some group methods and techniques.
4. "Effective" teaching.

Some perusal of the articles and books indicated in the following bibliography will give you an idea of what to expect in meetings on this topic.

Benne, Kenneth and Muntyan, Bozidar, *Human Relations and Curriculum Change*, Bulletin No. 7, Illinois Secondary School Curriculum Planning Program, Urbana, Illinois, 1949.

Bradford, Leland, editor, "Group Dynamics and Education," pamphlet series, *Journal of the National Education Association*, 1948–49.

Cantor, Nathaniel, *The Dynamics of Learning*, Buffalo, N. Y.: Foster and Stewart, 1946.

Cunningham, Ruth, *et al.*, "Getting the Group Habit," *Educational Leadership*, IV (March, 1947), 380–85.

Cunningham, Ruth, *et al.*, *Understanding Group Behavior in Boys and Girls*, New York: Teachers College, Columbia University, Bureau of Publications, 1951.

Jenkins, David, "What of Group Dynamics?" *Adult Education Journal*, IX (April, 1950), 54–60.

Jennings, Helen, *Sociometry in Group Relations. A Work Guide for Teachers.* American Council on Education, Washington, D. C., 1948.

Spiegel, Hans, "A Course in Human Relations: Some Group Methods and Techniques," *Adult Education Journal*, IX (April, 1950), 61–65.

Thelen, Herbert, "Educational Dynamics: Theory and Research," *Journal of Social Issues*, Vol. VI, No. 2, 1950.

Trow, William Clark, *et al.*, "Psychology of Group Behavior: The Class as a Group," *Journal of Educational Psychology*, XLI (October, 1950), 322–38.

"Two Lessons of Group Dynamics," *Educator's Washington Dispatch*, Deep River, Conn., 1948.

"We Look at Grouping," *Educational Leadership*, March, 1947 (entire issue).

Another very successful group at last summer's meeting was "Effective Speaking in the Classroom." This group used a tape recorder to enable people to listen to their own voices. Each member of the group addressed the group briefly. The leader—and the group—criticized each speaker and pointed out bad habits, (probably of many years' accumulation), and showed ways of correcting them. We are prone to ignore the immense importance of the use of the voice in the classroom. Why not have a series of meetings designed to do something about it? Effective teaching is by no means a function only of our knowledge of mathematics.

A third topic that cuts across elementary, junior high school, and senior high

school lines is "The Technique of Explanation." What could be more pertinent for mathematics teachers at all levels? Full information about this can be obtained by writing Edmund C. Berkeley and Associates, 36 West 11th Street, New York 11, N. Y.

Another very successful variation for one or more meetings is entitled "Swap Shop: How Do YOU Do It?" Scores of questions arise at every level of mathematics teaching. Share your ideas. This topic gives each member of the group a chance to talk—a refreshing change sometimes from constant lectures.

Closely allied to "The Swap Shop" in spirit but involving a more or less formal talk by an individual is the "How I Teach . . . " type of topic. With a skillful leader and one or two "primers" in the audience this kind of topic can usually evoke lively discussion.

Then, for pure fun and novelty try a "Carnival Night." We reproduce a description provided by The Association of Teachers of Mathematics in New England.

A committee arranged nineteen tables with a different problem, experiment, or puzzle to try. Each participant is assigned a starting table and progresses from table to table according to a special mathematical formula which is guaranteed to mix the crowd up well. A whistle blown at stated intervals sets the time for moving on. Here are some samples of what one finds.

1. A magic square to complete, 12 of its 49 members missing.
2. Several figures in which to count the number of squares, each one more confusing than the one before
3. Twenty-seven bottles filled with shot, one heavier than the others—a pair of scales—the problem to find that heavier one in three weighings
4. A table covered with regular polygons cut out of colored cardboard, with the problem of arranging them in repeating patterns
5. Directions for outlining curves of pursuit, using paper drinking cups
6. Measuring heights with a mirror—a moon and star fastened high on the wall to provide a problem
7. A pendulum fixed to make a pattern of fine sand as it swings over a square boxed in area on the floor
8. Tower of Hanoi
9. Vernier calipers to use in measuring small objects
10. Directions for using an alidade, and a sextant
11. Coin tossing test of the probability formula
12. Newspapers to tear and fold to see how many thicknesses can be torn or folded

An "Information Please" program can provide a very entertaining and profitable evening. Have a panel of five or six "experts" and ask your members to submit questions in advance of the meeting though questions raised at the meeting should be accepted also. This meeting goes better if the audience is given an opportunity and encouraged to participate with the experts.

If the group is small just an evening of shop talk is good, very informal and nothing planned in advance except for two or three "primers" ready to jump into prolonged silences.

NCTM Speakers Bureau

The Speakers Bureau is now ready to render service to interested Affiliated Groups. By September, it is anticipated that the scope and type of service will be materially expanded however. By that time, the Affiliated Groups of the northeast area will each have (1) a list of topics which the membership suggested for meetings and (2) a roster of speakers nominated by the membership. In turn, this information will be assembled and organized by the central Speakers Bureau and it will be available to interested Affiliated Groups.

Observations of the results of the pilot endeavors indicate that utilization of the materials collected improve meetings, increase attendance of the meetings and help build membership. It seems that the use of lay members' suggestions in planning meetings may be a clue to better meetings and to membership growth. Although difficult to substantiate statistically, it appears that the officers of associations may employ the Group Dynamics techniques used in developing this program to promote more popular meetings, and the retention and expansion of membership.

Although the Advisory Committee of

(*Continued on page 394*)

Program
Thirteenth Summer Meeting
The National Council of Teachers of Mathematics

Western Michigan College of Education, Kalamazoo, Michigan
August 23, 24, 25, 26, 1953

Host Organizations
Michigan Council of Teachers of Mathematics
Western Michigan College of Education

Convention Theme
Let's Look Ahead

SUNDAY, AUGUST 23

3:00–8:00 P.M. **Registration and Room Assignment**—Draper-Siedschlag Hall

MONDAY, AUGUST 24

8:00 A.M.–8:00 P.M. **Registration**—Draper-Siedschlag Hall.

8:00 A.M.–5:00 P.M. **Exhibits**—Administration Building[1]

8:45–10:00 A.M. **General Session**—Main Dining Room, Burnham Hall

What I See in the Future for Mathematics Education, E. H. C. HILDEBRANDT, Northwestern University, Evanston, Illinois

10:15–11:45 A.M. **Elementary Section**—Room 203[2]

Teaching Procedures Best Designed to Achieve Understandings in Arithmetic, GERALDINE KAUFFMAN, Public Schools, East Chicago, Indiana

Techniques for the Evaluation of Meanings in Arithmetic, ANN PETERS, Teachers College, Keene, New Hampshire

Integrating Arithmetic in the Elementary School Curriculum, MARGUERITE WOLFINGER, Wisconsin State College, Superior, Wisconsin

10:15–11:45 A.M. **Junior High Section**—Room 204

Meeting Individual Needs of Pupils in General Mathematics Courses, OLIVE LESKOW, Tolleston High School, Gary, Indiana

Activities Which Develop Power in Problem Solving, THEODORA NELSON, Nebraska State Teachers College, Kearney, Nebraska

Effective Utilization of Community Resources in Teaching Mathematics, STANLEY F. SCHENCK, Goshen High School, Goshen, Indiana

10:15–11:45 A.M. **High School Section**—Room 205

This section is sponsored by the Ohio Council of Teachers of Mathematics

Symposium: *Promoting Pupil Discovery*

Chairman: ONA KRAFT, Collinwood High School, Cleveland

Analyst: OSCAR SCHAAF, Ohio State University, Columbus

Participants: W. F. CORNELL, Bowling Green State University, Bowling Green; CLARENCE HEINKE, Capital University, Columbus; NELSON T. HOWE, Firelands Local Schools, Birmingham; JOHN F. SCHACHT, Bexley High School, Columbus; EUGENE SMITH, Ohio State University, Columbus

10:15–11:45 A.M. **College Section**—Room 206

Topic: *College Mathematics for the Non-Mathematician*

[1] The school exhibits will be located in the north corridor of the third floor while the commercial exhibits will be located in Room 208

[2] All of the section meetings, discussion groups, laboratories, exhibits and film showings will be held in the Administration Building. The general sessions, mixer, banquet and luncheon will be held in other locations as noted.

A Course in Mathematics for the Training of Teachers of Vocational Agriculture, JAMES J. MCLAUGHLIN, Wisconsin State College, River Falls, Wisconsin

Developing a Balanced Community College Mathematics Curriculum, JAMES L. SPENCELEY, Alpena Community College, Alpena, Michigan

The Evolution of a Basic Mathematics Course, HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan

10:15–11:45 A.M. **Affiliated Groups Section**—Room 210

Topic: *The Articulation of Mathematics Education with Business and Industry*

The Committee on Cooperation of Mathematics Education with Industry . . . A Progress Report, PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan

Mathematics in Industrial Research, ARVID ROACH, Research Laboratories Division, General Motors Corporation, Detroit, Michigan

Mathematics for Industry at the Technical Level, W. D. MERRIFIELD, Director of Industrial Education, Chrysler Corporation, Detroit, Michigan

10:15–11:45 A.M. **State Course of Study Section**—Room 108

Wyoming Course of Study in Arithmetic, JESSIE MAE HALSTED, The University of Wyoming, Laramie, Wyoming

Mathematics Curriculum Materials in Iowa, HAROLD C. TRIMBLE, Iowa State Teachers College, Cedar Falls, Iowa

1:30–2:45 P.M. **Elementary Section**—Room 203

Various Approaches to Measuring the Development of Number Abilities, SARA RHUE, University of Wisconsin, Madison, Wisconsin

Anticipating Learning Difficulties in Arithmetic, CHESTER MCCORMICK, Wayne University, Detroit, Michigan.

1:30–2:45 P.M. **Junior High Section**—Room 204

Significance of Pupil Discovery in Learning Arithmetic, IRVIN H. BRUNE, Iowa State Teachers College, Cedar Falls, Iowa

Factors Which Contribute to Success in Problem Solving, JACK V. HALL, Campus Elementary School, Iowa State Teachers College, Cedar Falls, Iowa

1:30–2:45 P.M. **Algebra Section**—Room 209

Teaching Properties of Whole Numbers as an Introduction to Algebra, JOHN A. BROWN, Wisconsin High School, Madison, Wisconsin

A Usable Set of Postulates for Ninth Grade Algebra, JOHN RECKZEH, State Teachers College, Jersey City, New Jersey

Debatable Issues in the Teaching of Algebra, HENRY A. MEYER, Central High School, Evansville, Indiana

1:30–2:45 P.M. **High School Section**—Room 205

The Teaching of Statistics in the General Mathematics Course of Senior High School, C. N. FUQUA, Champaign Senior High School, Champaign, Illinois

Helping High School Students Develop More Effective Study Habits in Mathematics, HENRY SWAIN, New Trier Township High School, Winnetka, Illinois

1:30–2:45 P.M. **Mathematics Applications Section**—Room 108

Topic: *Applications of Mathematics in Industry*, CARL J. CARLTON, Training Department, Ford Motor Company, Dearborn, Michigan

Applications of Mathematics Concepts to Science, HELEN S. HUGHES, Hudson High School, Hudson, Wisconsin

1:30–2:45 P.M. **Guidance Section**—Room 206

This section is sponsored by the Detroit Mathematics Club.

Topic: *Guidance in Mathematics in the Senior High School*,

Chairman: ELMER MCDAID, Department of Instructional Research, Detroit Public Schools

Participants: MARGARET HOLMAN, Chadsey High School; MARJORIE SMITH, Redford High School; WESLEY THOMPSON, Denby High School.

1:30–2:45 P.M. **Teacher Education Section**—Room 210

A Program for the Revision of Certification Requirements, JAMES H. ZANT, Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma

Realistic Programs for In-Service Training, DANIEL W. SNADER, University of Illinois, Urbana, Illinois

2:30–4:30 P.M. **Elementary Laboratories**—Rooms 320 and 321

Early Elementary: DOROTHY SCHERROCK, Marshall School, Detroit, Mich.

Later Elementary and Junior High: GERTRUDE SMITH, McKinstry School, Detroit, Michigan and SIGFRID ANDERSON, Jefferson School, Detroit, Michigan

2:30–4:30 P.M. **Geometry and College Mathematics Laboratory**—Room 329

ROBERT C. YATES, West Point Military Academy, West Point, New York

3:15–4:30 P.M. **Discussion Groups**

Group 1. Room 209

Topic: *What Experiences Build Arithmetic Readiness in the Early Grades?*

Leader: MARIAN BURTON, Wayne University, Detroit, Michigan

Group 2. Room 210

Topic: *An Exchange of Ideas for Programs for Mathematics Clubs and Assemblies*

Leader: DOROTHY RIDGWAY, Central High School, Ft. Wayne, Indiana

Group 3. Room 211

Topic: *What Can We Do to Make the Ninth Grade General Mathematics Courses More Popular?*

Leader: IZETTA SPARKS, North Texas State College, Denton, Texas

Group 4. Room 310

Topic: *What Should Be the Content of General Mathematics Beyond the Ninth Grade?*

Leader: DONALD MARSHALL, Dearborn High School, Dearborn, Michigan

Group 5. Room 311

Topic: *How Can the Classroom be Organized to Teach Widely Differing Ability Levels Effectively?*

Leader: GRACE ARBOGAST, Dennis Junior High School, Richmond, Indiana

Group 6. Room 312

Topic: *What Purposes of the Teaching of Geometry Should be Emphasized and Tested?*

Leader: VIRGINIA LEE PRATT, Central High School, Omaha, Nebraska

Group 7. Room 313

Topic: *Using Kodachrome Slides to Inform Parents About Modern Methods of Teaching Arithmetic*

Leader: DOROTHY WINTER, Burns School, Detroit, Michigan

Group 8. Room 314

Topic: *The Importance of Emphasizing Mathematical Relationships in Grades 7–9*

Leader: MARY C. NEVILLE, Michigan State Normal College, Ypsilanti, Michigan.

4:00–5:00 P.M. **Showing of Mathematics Films**

Elementary—Room 108

Secondary—Room 309

7:30–10:00 P.M. **Get-Acquainted Session**—Draper-Siedschlag Hall

Entertainment, games, square dancing

TUESDAY, AUGUST 25

8:00 A.M.–5:00 P.M. **Registration**—Draper-Siedschlag Hall

8:00 A.M.–5:00 P.M. **Exhibits**—Administration Building

8:45–10:00 A.M. **General Session**—Main Dining Room, Burnham Hall

The Implications of Large Scale Machine Computation for the Teaching of Mathematics, HARRY D. HUSKEY, Wayne University, Detroit, Michigan

10:15–11:45 A.M. **Elementary and Junior High School Section**—Room 108

Development of the Ability to Compute Mentally, HAROLD E. MOSER, State Teachers College, Towson, Maryland

Presenting Arithmetic as a System of Related Ideas, LAWRENCE SMITH,

Central Michigan College, Mt. Pleasant, Michigan

Increasing Pupil Participation in Mathematics Classes, JAY DITTMAN, Durfee School, Detroit, Michigan

10:15–11:45 A.M. **General Mathematics Section**—Room 203

This section is sponsored by the Wisconsin Mathematics Council.

Panel Discussion: *Mathematics for the Many*

Chairman: GRANT THAYER, Janesville High School, Janesville

Participants: MARJORIE DAVIS, Janesville High School, Janesville; LYDIA GOERZ, Mary D. Bradford High School, Kenosha; ANNA NEITZEL, Washington Park High School, Racine.

10:15–11:45 A.M. **Geometry Section**—Room 204

Topic: *What Approach Shall We Use in Teaching Geometry?*

Approach Through Critical Thinking, EUGENE P. SMITH, University School, Ohio State University, Columbus, Ohio

Approach Through Experimentation, EILEEN BECKETT, Lebanon High School, Lebanon, Indiana

Approach Through Use of Visual Aids, ALLENE ARCHER, Thomas Jefferson High School, Richmond, Virginia

10:15–11:45 A.M. **High School Section**—Room 205

The Mathematical Understandings of In-Coming College Freshmen and the Implications for the High School Teacher, EMALOU BRUMFIELD, Kent State University, Kent, Ohio

A Functional Mathematics Classroom—How Should It Be Planned and Equipped? JOY E. MAHACHEK, State Teachers College, Indiana, Pennsylvania

Integrating Meanings and Relationships in the Teaching of Algebra, W. H. EDWARDS, Central High School, Detroit, Michigan

10:15–11:45 A.M. **Gifted Student Section**—Room 210

Enrichment Activities in General Mathematics, EVELYN BONNINGTON, Redford High School, Detroit, Michigan

Student-Made Projects in Geometry, DON WEAVER, Elmhurst High School, Ft. Wayne, Indiana

Enrichment Activities in Algebra, WILLIAM POSORSKE, Rochelle Township High School, Rochelle, Illinois

10:15–11:45 A.M. **Teacher Education Section**—Room 206

Teacher Education in Mathematics: A Discussion of the Madison Symposium

Chairman: PHILIP PEAK, Indiana University, Bloomington, Indiana

Participants: H. D. LARSEN, Albion College, Albion, Michigan; P. D. EDWARDS, Ball State Teachers College, Muncie, Indiana; HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa; LAWRENCE WAHLSTROM, State Teachers College, Eau Claire, Wisconsin

1:30–2:45 P.M. **Elementary Section**—Room 203

Teaching the Meanings of Common and Decimal Fractions, FRANCES COCHRAN, Marr School, Detroit, Michigan

Generalizations in Teaching the Operations With Common Fractions, OPAL BASHELIER, Burt School, Detroit, Michigan

How I Teach Multiplication and Division of Decimal Fractions, OLIVE WEAR, Ft. Wayne Public Schools, Ft. Wayne, Indiana

1:30–2:45 P.M. **Junior High Section**—Room 204

How I Teach Ratio and Proportion, MILDRED LUSE, Hoagland School, Ft. Wayne, Indiana

Mathematical Recreations, MURIEL KILPATRICK, Lincoln Consolitated Training School, Ypsilanti, Michigan

Programs for Mathematics Clubs and Assemblies, DEBORAH JONES, Foch School, Detroit, Michigan

1:30–2:45 P.M. **High School Section**—Room 205

This section is sponsored by the Illinois Council of Teachers of Mathematics.

The Mathematics Program in the University High School of the University of Illinois, MAX BEBERMAN, University of Illinois, Urbana

1:30–2:45 P.M. **Algebra Section**—Room 206

Classroom Procedures Conducive to Learning in Algebra, CATHERINE A. V. LYONS, Perry High School, Pittsburgh, Pennsylvania

Contributions of Arithmetic in the Teaching of Algebra, WILBUR HARRIS, Findlay High School, Findlay, Ohio

1:30–2:45 P.M. **General Mathematics Section**—Room 210

Content of General Mathematics Beyond the Ninth Grade, LEE E. ARTHUR, Riverside-Brookfield High School, Riverside, Illinois

Consumer Education and the General Mathematics Courses of the Senior High School, GILBERT NELSON, Lincoln High School, Cleveland, Ohio

Mathematics and the Development of Good Citizens, CATHERINE D. MEEHAN, Port Huron High School, Port Huron, Michigan

2:30–4:30 P.M. **Algebra and General Mathematics Laboratory**—Room 320

Leader: LAUREN G. WOODBY, State Teachers College, Mankato, Minnesota

2:30–4:30 P.M. **Geometry Laboratory**—Room 321

Leaders: DON WEAVER, Elmhurst High School, Ft. Wayne, Indiana; and ELLI OTTESON, Eau Claire High School, Eau Claire, Wisconsin

3:15–4:30 P.M. **Discussion Groups**

Group 9. Room 209

Topic: *How Can Teachers in an Elementary School Coordinate Their Efforts to Improve Problem Solving Ability?*

Leader: ANNA MARIE EVANS, Cincinnati Public Schools, Cincinnati, Ohio

Group 10. Room 210

Topic: *How Can We Obtain Maximum Benefits From the Use of Film Strips in Teaching Arithmetic?*

Leader: GEORGE PARLATTO, Hosmer School, Detroit, Michigan

Group 11. Room 211

Topic: *Activities Which Promote Increased Success in Teaching Areas and Volumes in the Junior High School*

Leader: RAYMOND MORLEY, McMichael School, Detroit, Michigan

Group 12. Room 310

Topic: *The Syllogism in Plane Geometry*

Leader: JAMES F. ULRICH, Arthur Hill High School, Saginaw, Michigan

Group 13. Room 311

Topic: *Teaching the Trouble Spots in Algebra*

Leader: KATHARINE HILL, University High School, University of Michigan, Ann Arbor, Michigan

Group 14. Room 312

Topic: *The Need for More Effective Guidance in Mathematics*

Leader: HELEN B. DEJONGE, East Grand Rapids High School, East Grand Rapids, Michigan

Group 15. Room 314

Topic: *Paradoxes for Enrichment of High School Mathematics Courses*

Leader: KENNETH R. CONKLING, Frankton High School, Frankton, Indiana

Group 16. Room 313

Teaching the Function Concept More Effectively, ROBERT LANKTON, Mumford High School, Detroit, Michigan

4:00–5:00 P.M. **Showing of Mathematics Films**

Elementary—Room 309

Secondary—Room 108

6:15 P.M. **Banquet**—Main Dining Room, Draper-Siedschlag Hall

Encouraging Signs for the Teaching Profession

J. B. EDMONSON, Dean Emeritus, University of Michigan, Ann Arbor, Michigan

WEDNESDAY, AUGUST 26

8:00 A.M.–12:00 M. **Registration**—Draper-Siedschlag Hall

8:00 A.M.–5:00 P.M. **Exhibits**—Administration Building

8:45 –10:00 A.M. **General Session**—Main Dining Room, Burnham Hall

How Mathematics Functions in the Paper Industry, G. K. FERGUSON, President, Watervliet Paper Co., Watervliet, Michigan

10:15–11:45 A.M. **Elementary Section**—Room 203

Techniques for Determining Pupil Readiness for New Work, D. BANKS WILBURN, Marshall College, Huntington, West Virginia

Teaching Different Ability Levels in the Same Class, WILLIAM HIGGINS, Ball State Teachers College, Muncie, Indiana

Helping Slow Learners Achieve Success Experiences in Arithmetic, SINA MOTT, Southern Illinois University, Carbondale, Illinois

10:15–11:45 A.M. **Junior High Section**—Room 204

Topic: *What Activities Can We Use to Interest and Stimulate the Slow Learner*? Suggestions from Some Detroit Teachers

Chairman: LORANA TREMPER, Garfield School

Participants: LAURA LUHR, Mumford High School; LUCILLE MARTIN, McMichael School; GERTRUDE MCCONNELL, Foch School

10:15–11:45 A.M. **High School Section**—Room 205

The Use of Elementary Logic in the Teaching of Algebra and Geometry, FRANK B. ALLEN, Lyons Township High School and Junior College, LaGrange, Illinois

The Role of Vocabulary Building in the Teaching and Learning of Mathematics, SYLVIA VOPNI, University of Washington and Seattle Public Schools, Seattle, Washington

Evaluation of Teaching Materials for Algebra, NELLE KITCHENS, Hickman High School, Columbia, Missouri

10:15–11:45 A.M. **State Course of Study Section**—Room 108

A Course of Study in Mathematics for Secondary Schools in Pennsylvania, JOHN C. HOSHAUER, State Teachers College, Edinboro

Functional Mathematics in the Secondary School in Florida

W. A. GAGER, University of Florida, Gainesville, Chairman; CHARLOTTE CARLTON, Miami Beach High School, Miami Beach; F. W. KOKOMOOR, University of Florida, Gainesville; LILLA LYLE, Miami Beach High School, Miami Beach

10:15–11:45 A.M. **Geometry Section**—Room 210

Effective Classroom Organization for the Teaching of Geometry, KATHLEEN O'DONNELL, University of Kansas, Lawrence, Kansas

Teaching Plane Geometry as a Laboratory Subject, RODERICK MCLENNAN, Arlington Heights High School, Arlington Heights, Illinois

Experiments in Geometry, ETHEL NIEJAHR, Beaver Dam High School, Beaver Dam, Wisconsin

10:15–11:45 A.M. **College Section**—Room 206

Topic: *College-High School Articulation*

The Problem of Articulation with the High Schools, RALPH C. HUFFER, Beloit College, Beloit, Wisconsin

An Arithmetic Improvement Service for College Students, ISOBEL BLYTH, Michigan State College, E. Lansing, Michigan

Problems Connected with the Placement of High School Graduates in Collegiate Courses in Mathematics, H. P. EVANS, University of Wisconsin, Madison, Wisconsin

12:30–2:15 P.M. **Luncheon**—Main Dining Room, Draper-Siedschlag Hall

(Topic to be announced)

R. H. BING, University of Wisconsin, Madison, Wisconsin

ANNOUNCEMENTS

Registration: The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elemen-

tary schools. The fee for non-members and visitors is $1.50. Under-graduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, and members of the press, are not charged a registration fee, but should register.

You are urged to register in advance and requested to check in at the Registration Desk upon arrival at the meeting. Registration headquarters will be in the Draper-Siedschlag Hall.

For advance registration, use the Registration and Reservation form to be found on pages 393–94. Registrations and reservations received before August 5 will be acknowledged by return mail.

Accommodations in College Residence Halls: The Western Michigan College has made available its residence hall, the Draper-Siedschlag Hall, as living quarters for those attending Council meetings. Reservations may be made for four nights —from Sunday night, August 23, through Wednesday night, August 26.

Food Service: Meals will be served in the Draper-Siedschlag Hall. Meal service will begin with breakfast on Monday, August 24, and end with the luncheon on Wednesday, August 26. Meals will be served cafeteria style except for the banquet on Tuesday evening and the closing luncheon on Wednesday noon. No dinner will be served on Wednesday night, August 26.

Costs: The charge for meals and room for the complete conference will be $17.50 per person. This charge includes the banquet and the luncheon.

The room charge is based on the assumption that since the rooms are all double rooms and provided with twin beds, each room will be occupied by two persons. The management of the dormitory much prefers to have the rooms occupied in this way although it is not impossible for one to reserve for single occupancy. In that case, however, the charge for the rooms will be $3.00 per night instead of $2.00 per night and this would make the total $21.50 instead of $17.50.

Persons who find it impossible to arrive for registration on Sunday afternoon or who indicate at registration that they will not be able to remain for the entire conference will be charged for only such services as are included between the time of their registration and the time of their departure. It will not be possible, however, to obtain refunds for such things as the omission of a single meal. The charge will have to be a package charge which will be determined by the time of registration and the time of departure. It is hoped that most people will find it possible to arrive for registration on Sunday afternoon and that few will find it necessary to leave before the closing of the conference on Wednesday noon.

Reservations: Reservations for residence hall accommodations may be made by mailing the registration and reservation form not later than August 1 to Dr. Charles Butler, Department of Mathematics, Western Michigan College, Kalamazoo 45, Michigan. A deposit of $5.00 per person will be necessary for each application for reservation.

School Exhibits: There will be an exhibit of mathematical models, instruments, teaching aids, and other classroom materials. Teachers are invited to bring materials for the exhibit. Those who plan to bring materials from their schools should communicate in advance with Mr. Donald Worth, 715 West Walnut Street, Kalamazoo, Michigan, Chairman of the Exhibit Committee.

Many free and inexpensive teaching materials will also be on exhibit. Suggestions of recent materials which have become available and have classroom value should be sent to Mr. Alfred Capoferi, 19720 Meier Road, St. Clair Shores, Michigan.

Commercial Exhibits: Textbooks and commercial teaching aids will be on exhibit. Inquiries for exhibit space should be addressed to Dr. Charles Butler, Department of Mathematics, Western Michigan College, Kalamazoo, 45 Michigan.

Supplies and Equipment: Speakers and

ADVANCE REGISTRATION AND RESERVATION FORM

Please fill out completely and mail with remittance to Dr. Charles Butler, Department of Mathematics, Western Michigan College, Kalamazoo 45, Michigan, before August 1, 1953. Make check or money order payable to Dr. Charles Butler.

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Last Name First Name Initial

Address _____
Street and Number City Zone State

Registering as: Member of NCTM _____ Member of MAA _____ Non-Member _____
Elementary Teacher _____ Student _____ Exhibitor _____

Check your field of teaching interest: Elementary _____ J.H.S. _____ S.H.S. _____ Teacher Education _____ Supervision _____ Other _____

Room Reservations: A deposit of $5.00 per person is required.

_____ Myself only. I wish to room with _____

_____ Myself and wife or husband _____

_____ Myself and the following members of my family. Give ages of children: _____

_____

and other participants in the program who need blackboards, felt boards, projection equipment, or other materials should communicate not later than August 5 with Dr. Charles Butler, Department of Mathematics, Western Michigan College, Kalamazoo 45, Michigan.

Films and Film Strips: Persons wishing to view specific films while in attendance at the meeting should communicate with Mr. Ford Lemler, Audio Visual Education Center, University of Michigan, Ann Arbor, Michigan, not later than July 1.

Mail and Telegrams: Mail and telegrams for those attending the meetings should be addressed in care of the National Council of Teachers of Mathematics, Draper-Siedschlag Hall, Western Michigan College, Kalamazoo 45, Michigan.

Certificate of Attendance: If you care to take back to your school authorities a statement certifying your attendance at the convention, make request for it at the Registration Desk on Wednesday, Aug. 26.

Mathematics Laboratories: Five mathematics laboratories are planned. The plan of operation will include about a thirty-minute talk and demonstration on the construction and use of learning aids, models, etc., in the classroom. Each member of a laboratory group will be given an opportunity to make several models, for which a minimum cost charge for materials used will be made. Refer to the program for further information. Since the number who can attend any one of these sessions is very limited, advance registration is strongly advised.

Recreational Facilities: The recreational facilities of Western Michigan College will be available to everyone attending the conference and members of their families. The college owns a nine-hole golf course which is very near to the headquarters for the meeting and which will be available to any participant of the conference at a nominal cost.

Sight-Seeing Trip: A sight-seeing trip to the Upjohn Company's large penicillin plant has been arranged for Wednesday afternoon, August 26.

Note: For points not covered in this announcement, please write directly to Dr. Charles Butler, Department of Mathematics, Western Michigan College, Kalamazoo 45, Michigan.

ADVANCE REGISTRATION AND RESERVATION FORM

(*Continued from reverse side*)

Amount Enclosed for Room Reservations: $_____

Registration Fee: $0.50 or $1.50 $_____

Total Amount Enclosed $_____

Registration for Mathematics Laboratories:

Monday, Early Elementary _____

Monday, Later Elementary and Junior High _____

Monday, Geometry and College Math. _____

Tuesday, Algebra and General Math. _____

Tuesday, Geometry _____

Reservations for trip to Upjohn penicillin laboratories _____
If you have transportation there will be no charge for this trip.

Members of families and friends of those attending the meetings are welcome at all social and recreational events.

Program Committee

H. C. Christofferson, Miami University, Oxford, Ohio; Harold P. Fawcett, Ohio State University, Columbus, Ohio; Alice M. Hach, Slausson Junior High School, Ann Arbor, Michigan; Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland; Arvid Jacobson, Wayne University, Detroit, Michigan; Phillip S. Jones, University of Michigan, Ann Arbor, Michigan; John R. Mayor, University of Wisconsin, Madison, Wisconsin; Robert L. Morton, Ohio University, Athens, Ohio; Philip Peak, Indiana University, Bloomington, Indiana; Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey; C. L. Thiele, Detroit Public Schools, Detroit, Michigan; Henry Van Engen, Iowa State Teachers College, Cedar Falls, Iowa; Marie S. Wilcox, George Washington High School, Indianapolis, Indiana; and Irene Sauble, Detroit Public Schools, Detroit, Michigan, Chairman.

General Committees

General Chairman

Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan.

Exhibit Committee

Donald Worth, Kalamazoo, Michigan; Henry Wholihan, Mackenzie High School, Detroit, Michigan.

Affiliated Groups

(*Continued from page 385*)

the Speakers Bureau will be pleased to serve the officers of any Affiliated Group by submitting lists of topics and recommended speakers, the Committee is equally interested in sending suggestions for the organization of a local Speakers Bureau.

To Mary C. Rogers, the Advisory Committee extends due recognition for the organization of the Bureau and for the impetus to attempt the pilot programs. For the recognition of the possibilities inherent in the plan; for assistance in promoting the program, we respectfully acknowledge Howard F. Fehr.

HUBERT B. RISINGER, *Chairman*
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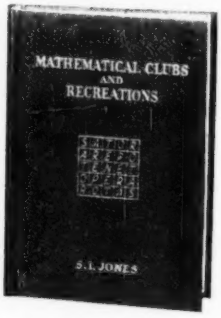

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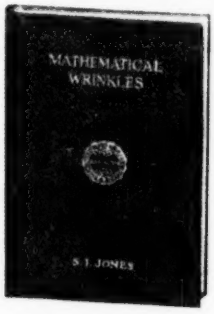

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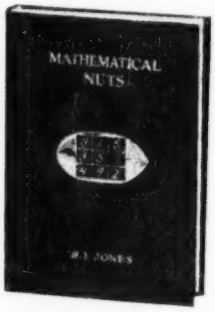

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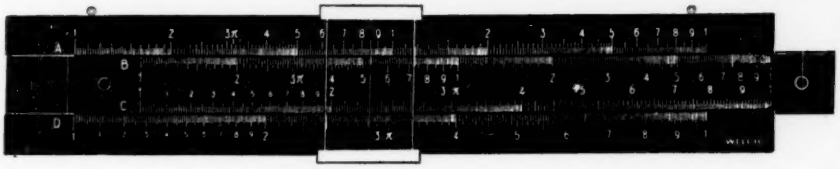

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